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Sedimentation analysis of selected Iowa lakes

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Sedimentation analysis
of
selected Iowa lakes

by

Umesh Gajanan Shetye

A Thesis Submitted to the
Graduate Faculty in Partial Fulfilment of the
Requirements for the Degree of

MASTER OF SCIENCE

Department: Civil and Construction Engineering
Major: Geotechnical Engineering

Approved:

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OBJECTIVE

The purpose of this study is to compare and contrast the sedimentation rates and the sediment distribution pattern in some selected Iowa lakes. The lakes that were chosen for this study are Pine Lake in Hardin County, Union Grove Lake in Tama County and Black Hawk Lake in Sac County. Pine Lake and Union Grove Lake are river dammed lakes while Black Hawk Lake has a glacial origin.

Empirical methods proposed to compute the amount of sedimentation and some of the proposed analytical models for sediment distribution have been reviewed.

Finally a novel approach to estimate the amount of sedimentation in lakes based on limited field data has been proposed and has been applied to the lakes under study.

CHAPTER 1: INTRODUCTION

1.1: Statement of the Problem

Construction of a dam across a stream alters its natural equilibrium by changing the characteristics of its discharge and the sediment transport capability. On coming in contact with the virtually stagnant water in a reservoir, the inflowing water velocity decreases thereby causing deposition of sediments. Reservoirs formed by constructing a dam across a flowing stream will, to some degree, be subjected to sedimentation.

Reservoirs are constructed for purposes which include among other things: water supply, irrigation water, hydropower, flood prevention and recreation. Reservoir sedimentation may seriously affect the purpose for which the reservoir was constructed; it is therefore necessary to estimate the rate of sedimentation and the useful life of the reservoir.

Problems associated with sedimentation of reservoirs include one or more of the following:

- Aggradation of upstream flood channels which may cause an increased frequency of flooding
- Increased costs associated with dredging of reservoirs
- Loss in the recreational value of the reservoir

The sediment distribution pattern in a reservoir affects decisions regarding the placement of sluices in dam walls and estimation of excess pressure on the dam due to the deposited sediments. Thus in addition to predicting the amount of sedimentation, it is also desirable to predict the spatial distribution of sediments in a reservoir. Although this research study does not aim at predicting the spatial distribution of sediments, some of the proposed mathematical models for spatial distribution to have been reviewed in Chapter 2 of the thesis.

1.2: Approach to the problem

One approach for engineers to estimate the amount of sedimentation in a reservoir is to use the Universal Soil Loss Equation, sediment delivery ratios, and trap efficiency curves. This approach is considered in detail in Chapter 2 of the thesis. Alternatively, the amount of sedimentation can be calculated by conducting sedimentation analysis of the lake using lake bathymetric maps from previous surveys or by probing sediment thicknesses. Future sedimentation rates can then be estimated from these historical data. Sedimentation analysis of the three lakes under study was conducted using SURFER, a computer software package.

It was determined that the lakes under study had characteristic normalized elevation vs normalized volume (NENV) curves. Normalization is a process wherein the variables, in this case the elevation and the volume, are represented as a percentage (ranging from 0% to 100%) of the maximum value of the variable. The NENV curves were used to propose a model (NENV model) to estimate the sedimentation in reservoirs without actually having to conduct extensive reservoir surveys. This approach is described in detail in Chapter 5 of the thesis.

CHAPTER 2: LITERATURE REVIEW

2.1: Introduction

Sedimentation analysis of a reservoir involves calculating the amount of sediments deposited in the reservoir. This can be achieved by using empirical relationships or conducting actual reservoir bottom surveys. Extensive reservoir survey is not quite feasible for very large reservoirs and in these cases empirical relationships are used to estimate the amount of sedimentation. These empirical relationships have been reviewed in the following sections. Also, some mathematical models proposed to explain the spatial distribution of sediments in reservoirs have been reviewed.

Sedimentation analysis of a reservoir essentially involves two components:

- Estimation of the amount, either on weight or volume basis, of sediments entrapped in the reservoir, and
- Spatial distribution of the deposited sediments within the reservoir

Estimating the amount of sedimentation basically consists of the following two parts:

- Estimate of the sediment inflow into the reservoir, and
- Estimate the amount of sediment that is actually retained by the reservoir

2.2: Sediment inflow estimation

Nearly all of the sediment transported to a reservoir by inflowing streams owes its existence to sheet and rill erosion of the soils on the watershed. Only a fraction of the total erosion is due to gully and stream channel erosion. Horton (1941) estimates that about 99% of the total erosion in the evolution of a drainage basin takes place by sheet and rill erosion on the watershed. Watershed characteristics which influence the volume of inflowing sediments are discussed in Chapter 3 of this thesis.

The gross erosion on a watershed is influenced by various inter-related factors. These factors include (Laronne and Mosley, 1982):

- relief of the watershed
- length of the slope
- climate
- conservation practices employed
- soil type

Schumm and Hadley (1961), as quoted in Schumm (1963), state that the sediment yield rates are an exponential function of the relief-length ratio of the watershed. This is as shown in Figure 2-1. It should be noted that the figure is plotted on a semi-logarithmic scale.

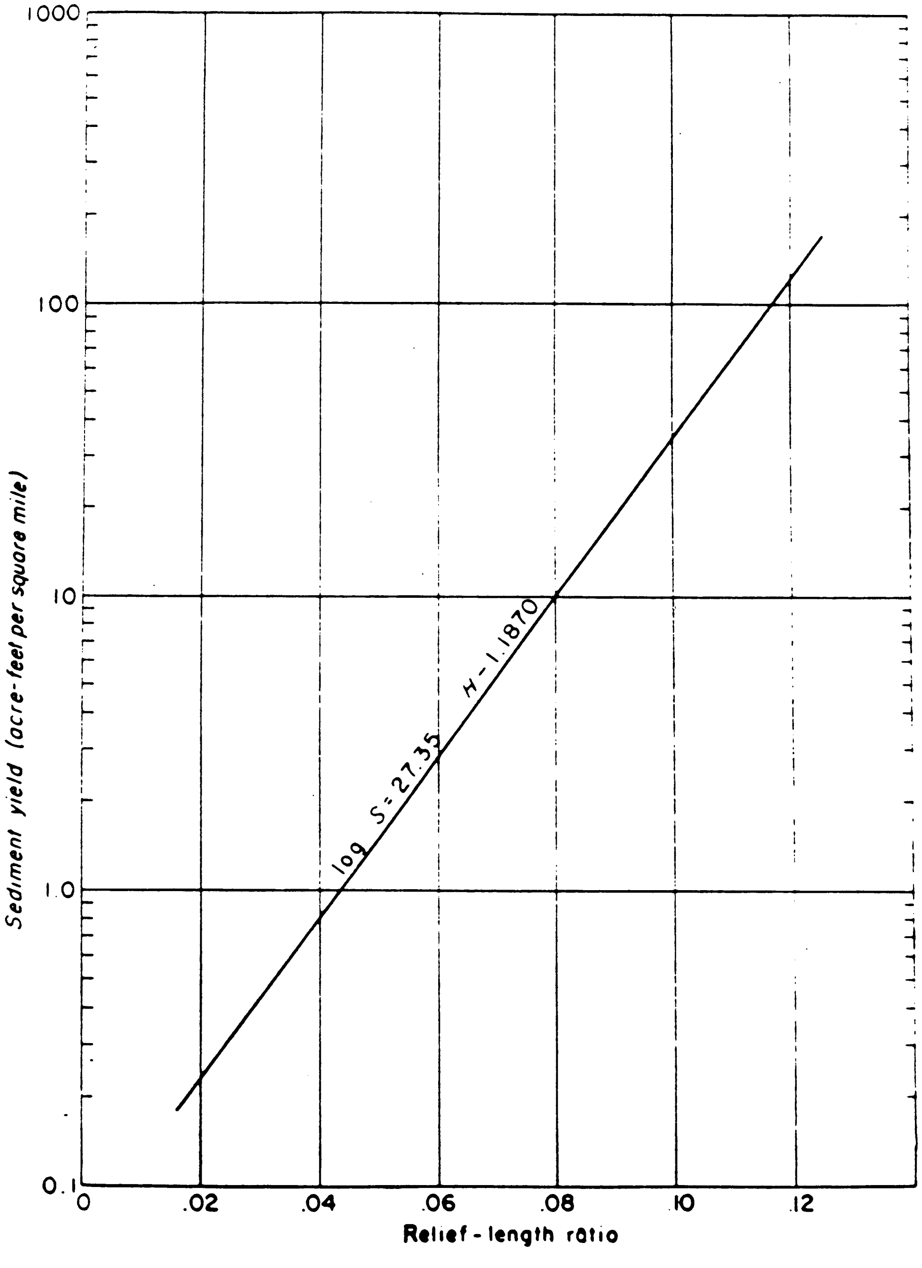

Figure 2-1: Relationship of sediment yield rates to relief-length ratio (Schumm, 1963)

The soils that are eroded are transported by the river/stream to the reservoir either as bed load or in suspension, depending on the particle size of the sediments. The amount of the eroded sediments that actually find their way to the reservoir can be estimated using any of the following approaches:

Sediment delivery ratio method

The percentage of sediment delivered from the erosion source to any specified downslope location is affected by such factors as the size and texture of erodible material, climate, land use, local environment, and general physiographic position (Vanoni,1975). The sediment delivery ratio (D), may be defined as the ratio of the sediment yield at the measuring point (Y), to the total material eroded from the watershed and the drainage system upstream from the measuring point (T).

$$D = Y/T$$

Usually, the sediment delivery ratio decreases with increasing drainage area in a basin that is relatively homogeneous with respect to soils, climate, and topography, but large downstream increases in erosional rates in a nonhomogeneous basin can increase the delivery ratio. Sediment delivery ratios can be developed from sediment yields obtained from reservoir surveys or by measurements at suspended load stations in comparison with erosion on the watershed (Taylor, 1970; Vanoni, 1975). Gottschalk and Brune (1950), as quoted in Vanoni (1975), have developed relationships for loess hills of Iowa and Nebraska and these are summarized in Figure 2-2.

The gross erosion on the watershed (T), is determined using the Universal Soil Loss Equation. The Universal Equation estimates the annual soil loss of a watershed (A) by taking into account numerous factors (Taylor, 1970). These include:

- rainfall factor (R)
- soil erodability factor (K)
- slope length (L) and slope steepness factors (S)
- cropping management factor (C)
- supporting conservation practice factor (P)

Then,

$$A = RKLSCP$$

The predicted annual soil loss has units used for K, the soil erodability factor, usually tons/acre. The other factors are dimensionless (Toy, 1977).

Sediment transport relationships

Various empirical and theoretical sediment transport models have been proposed by various researchers to explain the sediment transport phenomena in streams. These relationships are used to calculate the rate and quantity of sediment movement, and thus

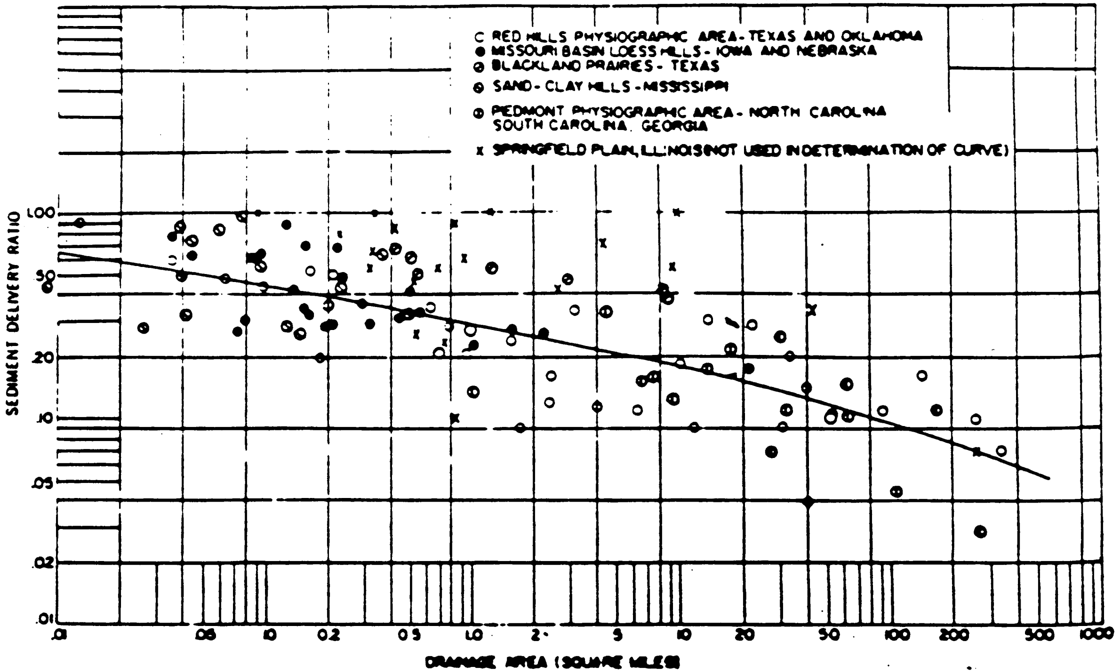

Figure 2-2: Relationship of sediment delivery ratio and drainage area (Gottschalk and Brune, 1950; From: Vanoni, 1975)

the inflowing sediment rate to the reservoir. The most commonly used sediment transport theory is the Einstein's bedload function, with one of its many modifications (Lopez, 1978).

Sediment yield rate curves

It is possible to predict the sediment yield of a watershed by comparing it with other watersheds having similar climatic, topographical and geologic characteristics. With drainage basin size as the independent variable it is then possible to construct sediment yield rate curves for similar reservoirs (Lopez, 1978). However the applicability of this method of determining sedimentation depends upon the skill of the engineer or the sedimentologist conducting the reconnaissance survey and hence this method is best applied for a preliminary estimate of the sediment yield of the watershed.

2.3: Sediment retained by the reservoir

All the sediment that finds its way to a reservoir is not retained by it. Some part of the inflowing sediment is lost when the water in the reservoir discharges over the spillway during periods of high discharge. Trap efficiency of a reservoir is defined as the ratio of the quantity of deposited sediment to the total sediment inflow. Trap efficiency can be attributed to the following factors:

- Sediment particle fall velocity which primarily depends on the shape and size of the particle
- Size and age of the reservoir, and
- Type of outlets and the operation of the reservoir

During periods of high inflow when there is an appreciable velocity of flow through the reservoir the inflowing sediment may be transported through the reservoir resulting in a lower trap efficiency.

For large reservoirs, with storage capacity above 10,000 acre-ft, the trap efficiency can be assumed as 100% (Vanoni, 1975). For smaller reservoirs the trap efficiency can be estimated using trap efficiency charts constructed based on measurement of sediment deposits in a large number of reservoirs.

Churchill (1948) presented a relationship to estimate the trap efficiency based on his research on Tennessee Valley Authority reservoirs. Churchill's method relates the percentage of incoming sediment passing through the reservoir and the sediment index. The sediment index is the ratio of the period of retention (capacity, in cubic feet, at mean operating pool level divided by the average daily inflow rate, in cubic feet per second) and the mean velocity (in feet per second, obtained by dividing average gross-sectional area, in square feet, into the inflow). The proposed relationship is as shown in Figure 2-3. According to Borland (1971), as quoted in Heinemann (1982), Churchill's method is more applicable than Brune's method for estimating trap efficiencies for desilting and semidry reservoirs.

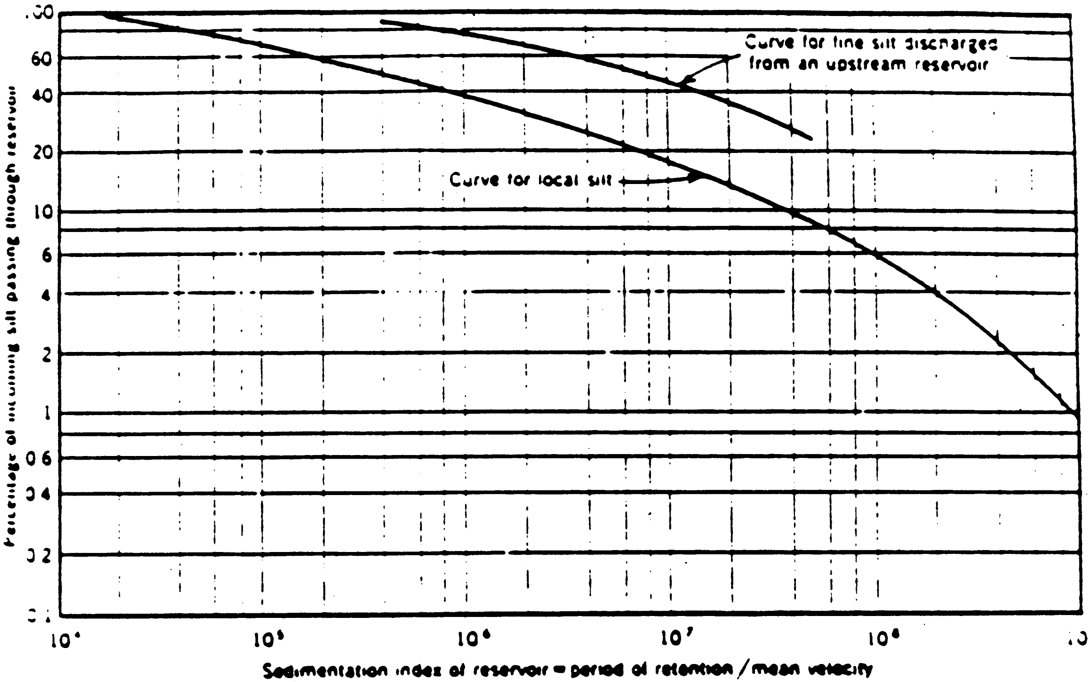

Figure 2-3: Reservoir trap efficiency curve, Churchill (1948)

It should be noted that Churchill's trap efficiency curve for a reservoir is different for local silt and for silt discharged from an upstream reservoir. The curve for local silt is used when the eroded soils are not trapped by an upstream reservoir on the watershed. If, however, such an upstream reservoir which acts as a silt trap for the lower reservoir exists then the upper trap efficiency curve as shown in Figure 2-3 is used.

Trap efficiency of small reservoirs can also be estimated using empirical relationships proposed by Brune (1953), based on the records of 44 normally ponded reservoirs. Brunes' curves relating trap efficiency and the ratio between the reservoir capacity, in acre-ft, and mean annual water inflow, in acre-ft, are shown in Figure 2-4. Brune's curves have been used more widely than other methods for estimating the trap efficiency of reservoirs (Heinemann, 1982). According to Gottschalk (1965), Brune's curves overestimate the trap efficiency. He based his conclusion on the basis of his studies conducted on 18 small reservoirs, wherein the measured trap efficiencies fell between or below Brune's envelope curves. The soils on the watershed of the reservoirs studied by Gottschalk were mostly composed of finer materials.

The "Committee on Sedimentation Engineering" (1975) and "National Engineering Handbook" of the Soil Conservation Service (1983) suggest the use of Brune's trap efficiency curves to estimate the trap efficiency of a reservoir. Based on his studies Brune developed upper and lower trap efficiency curves and the reservoirs studied by him had trap efficiency in this range.

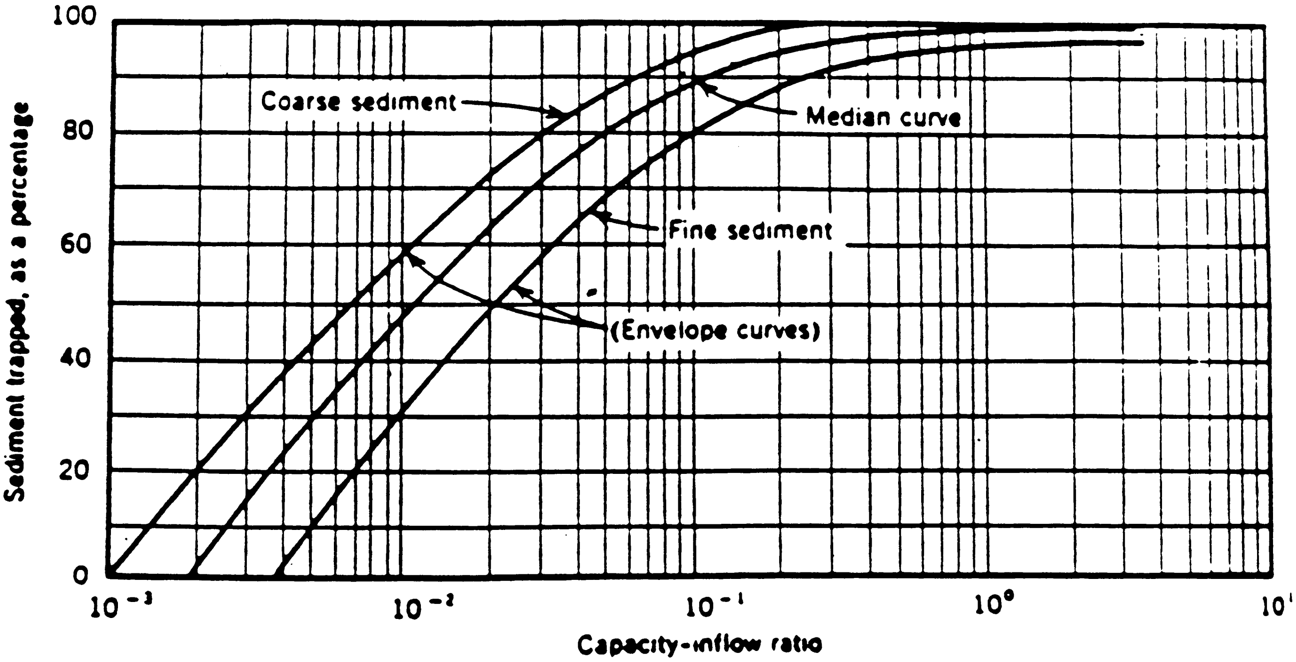

Figure 2-4: Reservoir trap efficiency curve, Brune (1953)

Chen (1975) has developed a series of curves for various particle sizes, relating trap efficiency to the ratio of basin area to outflow rate. According to him Churchill's curves and Brune's curves for determining the trap efficiency of reservoirs are compatible in the silt range. From the studies conducted by him he concludes that both Churchill's and Brune's trap efficiency curves tend to underestimate trap efficiency for coarser materials and overestimate it for finer materials. Thus, Gottschalk (1965) and Chen (1975), both conclude that Brune's curve overestimates the trap efficiency of a reservoir for finer materials. Chen's findings summarized in form of a graph are as shown in Figure 2-5.

Analytical methods that are available for estimating trap efficiency of reservoirs are based primarily on a function of the ratio of reservoir volume to inflow rates. Karaushev (1966) and Borland (1971), as quoted in Lopez (1978), have proposed equations to estimate trap efficiency. However, these methods do not include an analysis of sediment characteristics and hence are not widely used by sedimentation specialists (Lopez, 1978).

2.4: Distribution of sediment deposits

When a stream enters a reservoir, the velocity of the inflowing water decreases as it comes in contact with the water in the reservoir. The sudden reduction in the stream velocity results in the deposition of coarser sediments, like sands and silty sands, near the

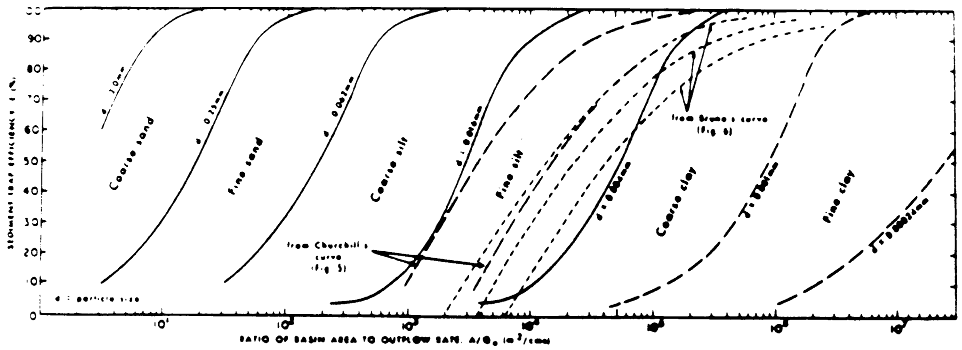

Figure 2-5: Trap efficiency expressed as a function of ratio of basin area to outflow rate (Chen, 1975)

mouth of the reservoir. This deposition continues until, at some distance within the reservoir, the flow velocity has been sufficiently reduced so that all the sediments of sand size or larger are deposited. The finer sediments which essentially consist of silts and clays are transported into the reservoir beyond the delta and are deposited throughout the bottom of the reservoir. Some of these bottom sediments are carried further down by density currents. A density current may be defined as the movement of a stream of fluid under, through, or over another fluid, the density of which differs by a small amount from that of the primary current. The most important and interesting type of density current, in lake sedimentation studies, is one which is formed by suspension of sediments. This type of density current is called as turbidity current. Turbidity currents composed of clay and silt sediments sometimes play an important role in the sediment distribution pattern of reservoirs (Middleton, 1966). A density current may also be generated in a reservoir due to the temperature difference in successive levels of water in the reservoir. This type of density current does not play a significant role in sediment distribution in a reservoir (Middleton, 1966).

Some of the factors that influence the mode in which the sediment is deposited through the reservoir include the size and texture of sediment particles, size and shape of the reservoir, reservoir inflow-outflow relations and the reservoir operating rules (Borland, 1958).

In a relatively narrow reservoir, in which the flow can spread evenly across the pool, the coarse sediments spread to form a delta. On the other hand, if the stream enters a wide pool, the flow tends to enter the pool as a jet, and a finite velocity of flow will continue along this line for a appreciable distance (Vanoni, 1975).

Figure 2-6 and Figure 2-7 illustrate the deposition of sediments in a reservoir. The coarser material gets deposited near the mouth of the reservoir (Figure 2-6). The finer sediments are carried further down the reservoir. As the delta formation takes place the successive stages are deposited further away from the mouth of the reservoir. Figure 2-7 shows the effect of density currents on the movement of sediments in the reservoir.

Actually the manner in which the sediments deposit depend on a number of interrelated factors which tend to affect the process and modify the magnitude and location of deposits. These factors include among others (Lopez, 1978).

- Fluctuating waterlevels
- Temporal variations in sediment and flow discharge
- Shape of the basin

Fluctuating water levels occurring in many reservoirs play an important role in delta formation. Usually, deltas tend to form at water surface elevation. At low elevations of the water surface, the deltas form far down in the reservoir. At higher water elevations they form near the upper reaches of the reservoir. Lowering of the water elevation causes the deltaic sediments to be carried and deposited downstream.

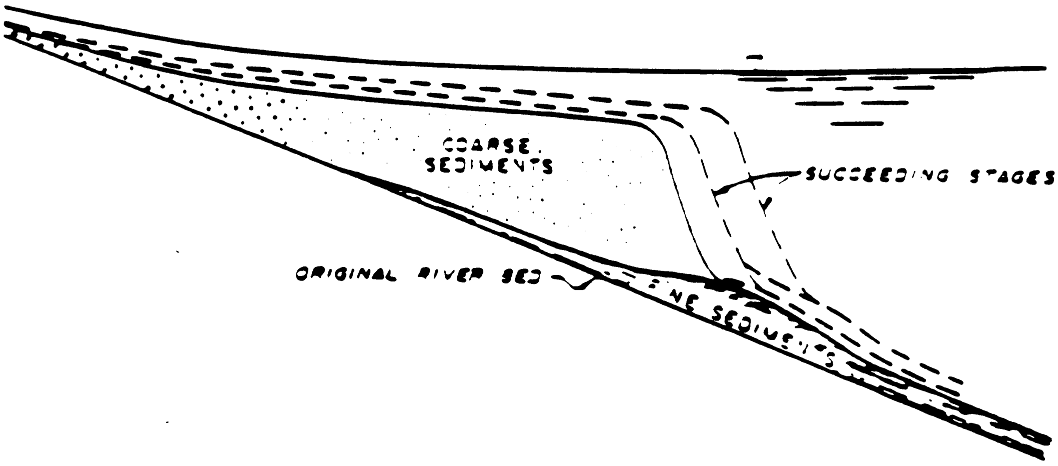

Figure 2-6: Typical reservoir-delta profile (Vanoni, 1975)

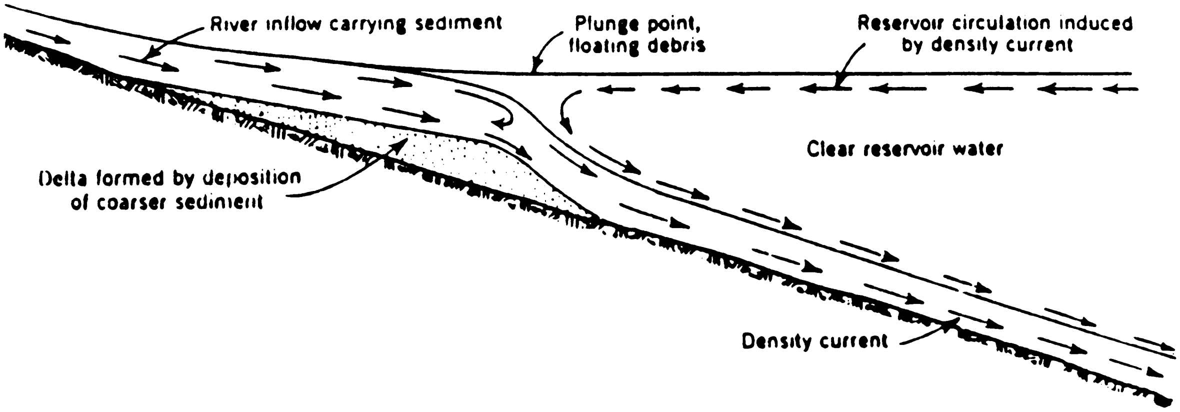

Figure 2-7: Effect of density currents (Vanoni, 1975)

Temporal variations in flow and sediment discharge also affect the process of delta formation. At the peak of the flow hydrograph, the sediment transport capability of a stream increases due to an increases in the velocity of flow. If this increase in the sediment transport capacity of the stream is greater than the corresponding inflowing sediment load, degradation of the bed occurs. This causes the delta to move downstream.

Shape of the basin also influences the manner in which the sediment is distributed spatially through a reservoir. The deposits normally will spread uniformly along the axis, for a regularly shaped reservoir. If the reservoir is irregular in shape, there might be marked irregularities in the depositional pattern (Lopez, 1978).

2.5: Mathematical models in reservoir sedimentation

More than twenty methods, empirical as well as analytical, have been proposed to calculate sediment distribution pattern in reservoirs (Annandale, 1987).

Prior to 1953, it was believed that the sediment discharging into a reservoir was transported to the dam wall and then was deposited from the lowest elevation upwards. Cristofano (1953), as quoted in Annandale (1987), was perhaps the first researcher to recognize that this was not true and to propose a model which took into account sediment distribution throughout the reservoir. Five years later, Borland and Miller proposed an empirical method, known as the area-reduction method, to explain the sediment distribution in a reservoir. Menne and Kriel (1959), Hobbs (1969), Borland (1970), Szechowycz and Qureshi (1973), Croley et al (1978), Pemberton (1978) and Chien (1982) have proposed empirical methods to explain the distribution of sediments in reservoirs (Annandale, 1987).

With the general availability of high speed digital computers, numerous models based on mathematical equations were proposed to explain the sediment distribution pattern in reservoirs. These models are based either on the diffusion and jet theory or sediment transport theories.

Jet and Diffusion Theory

The diffusion and jet theory assumes that the sediment particles travel as a submerged jet. A submerged jet differs from a free jet in two aspects

- Lack of gravitational influence
- Interaction between the jet and the surrounding fluid

Most of the researchers who have worked on the problem of sedimentation distribution assume that the jet spreads at a linear rate and the cross-sectional area of flow increases linearly downstream. Actually the boundary of the jet is curvilinear parabolic instead of linear with a triangular velocity distribution in the zone of diffusion (Lopez, 1978). Jet diffusion, as proposed by Lopez, can be schematically represented as shown in Figure 2-8.

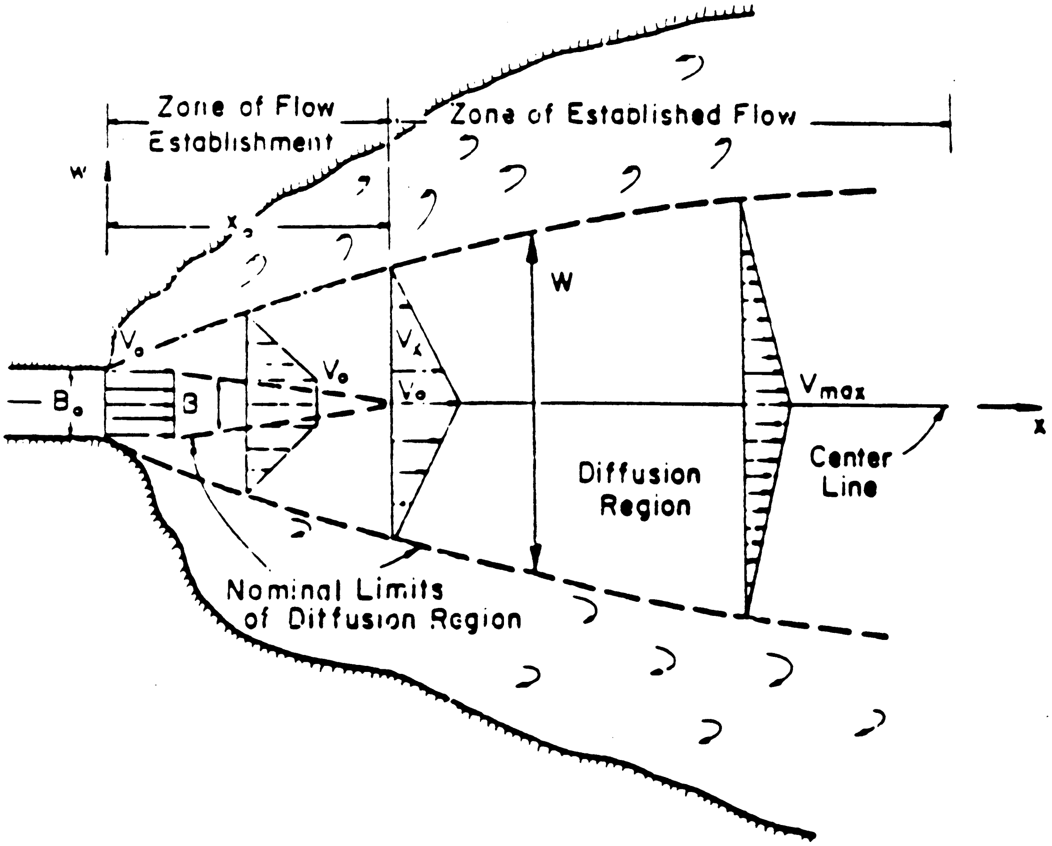

Figure 2-8: Jet Diffusion as proposed by Lopez

In most of the analytical models that have been proposed to explain the distribution of sediments, the major drawback is the large number of parameters that require calibration. In short, this means that the user of the models should have a fairly good understanding of the sediment distribution pattern before actually using the model that simulates sedimentation.

Chang and Richards model

Chang and Richards (1971) proposed a model that simulated the sediment distribution pattern in a reservoir for variable length and time. The equations used by them to simulate sediment distribution are given below.

Continuity of sediments

$$\frac{\partial}{\partial x}(vbhc)+\frac{\partial}{\partial t}(bhc)+p\frac{\partial}{\partial t}(bz)-q_s=0$$

Continuity of water

$$\frac{\partial}{\partial x}(vbh)+\frac{\partial}{\partial t}(bh)+\frac{\partial}{\partial t}(bz)-q_m=0$$

Dynamic Equation

$$\frac{\partial}{\partial t}(\rho_m vbh)+\frac{\partial}{\partial x}(\rho_m v^2bh)=-\frac{\partial}{\partial x}(\rho_m gb\frac{h^2}{2}))-\rho_m gbh[\frac{\partial Z}{\partial x}+\frac{\partial z}{\partial x}]-bz_0$$

where,

x = distance along the channel

v = mean velocity

h = mean depth of flow

b = channel width

z = thickness of the sediment layer

p = volume of sediment in unit volume of sediment layer

q_s = lateral discharge of sediments per unit length of channel

q_m = lateral discharge of sediment laden water per unit length of channel

c = the concentration of sediment in sediment laden water

g = acceleration due to gravity

Z = elevation of original bed from arbitrary datum

z_o = boundary shear stress

ρ_m = density of sediment laden water

In deriving these basic differential equations the following assumptions were made (Chang and Richards, 1971):

- The velocity is fairly uniform over the cross section
- The sediment-laden water is substantially homogeneous
- The channel is a wide rectangle in cross section and is assumed to be sufficiently straight and uniform in reach to permit mathematical representation by a one dimensional model.

In actuality, these assumptions do not strictly hold up to a real world problem. The velocity is not fairly uniform as is seen in Figure 2-8. Also, as the reservoir bottom is irregular, and as the sides have slopes, the channel is not a wide rectangle and uniform in reach in cross section. However, to permit mathematical representation by a one dimensional model these assumptions are essential.

As b, the channel width is assumed to be non-variable, there are four unknown variables, v, h, z and c (as defined previously), which have to be determined to explain the distribution of sediments in one-dimension. Another equation that can be used to solve for these unknowns, and used by Chang and Richards is given below

$$c=Kv^{m}h^{n}$$

where,

$$K=\frac{k}{gw}$$

In the above expression,

k = coefficient of sediment transport capacity

w = settling velocity of the particle

m, n = dimensionless parameters

Lopez's model

Perhaps the most detailed model yet to explain the spatial distribution of sediments in a reservoir was proposed by Lopez in 1978. His model not only simulates sediment distribution in the longitudinal direction but also takes into account the distribution of sediments in a transverse direction.

The advantage of Lopez's model over Chang and Richard's model is that Lopez does not assume a fairly uniform velocity over the cross section but a more realistic velocity distribution as shown in Figure 2-8 (Annandale, 1987). Also, instead of assuming the flow in the entire channel to be one dimensional, Lopez divided the flow in the reservoir into a number of imaginary channels, and the flow in channel was considered as one dimensional.

For the sake of modelling, Lopez divided the reservoir into three zones- river zone, transition zone and reservoir zone. The discharge of sediments in the river zone (primarily due to backwater effects), was viewed to be one dimensional. The flow in the transition zone was viewed to be two dimensional, and the flow pattern was assumed to be adequately characterized by the jet theory. The flow in the reservoir zone was divided into a number of imaginary canals, and the flow in each canal was considered to be one dimensional. The river reservoir system according to Lopez is as shown in Figure 2-9.

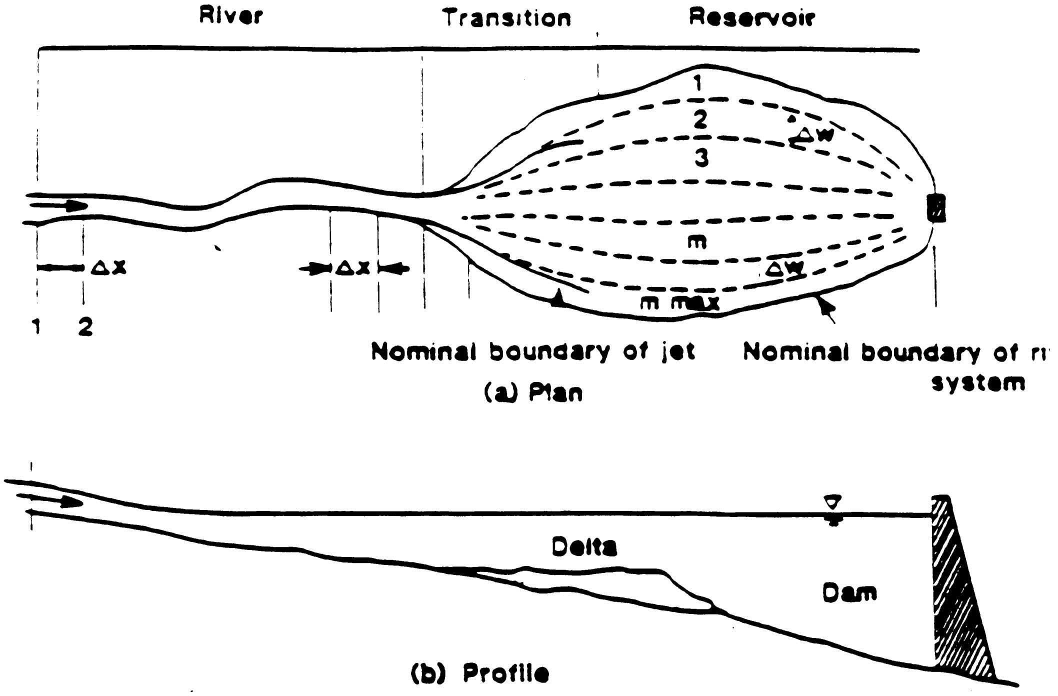

Figure 2-9: Reservoir-river system according to Lopez (1978)

2.6 Time independence concept and stream power theory

Erosion, sediment yield, and landscape formation are closely interrelated. Wolman and Miller (1960) have suggested that a river channel achieves a time-independent form that rapidly adjusts to the changing environmental conditions. The concept of time independence and dynamic equilibrium are closely tied up with the view of the landscape as an open system through which energy and matter are recycled (Laronne and Mosley, 1982).

Stream power theory can be used to explain the time independence concept and the dynamic equilibrium of the system by considering the reservoir as an open system. In a reservoir stable conditions occur when the applied stream power is minimized (Chang, 1979; Yang, 1976; as quoted in Annandale, 1987). Under such conditions uniform flow develops, sediment concentration remains constant, and the bed profile does not change with time. Unstable conditions on the other hand are characterized by continuously changing flow conditions and bed profile. Under unstable conditions, the applied stream power approaches a constant minimum value throughout a non-equilibrium system when stable conditions are approached (Annandale, 1987).

Stream Power Theory

Stream power concepts can be used to explain a variety of sediment transport phenomena, including those occurring in reservoir sedimentation (Annandale, 1987). When a sediment laden stream inflows into a reservoir, the distribution of the rate of internal entropy production is not uniform. The highest rate of internal entropy production occurs at the inflow into the reservoir, where a high degree of turbulence develops, as the flow is suddenly retarded by the virtually stagnant water in the reservoir. As the inflowing water moves through rest of the reservoir, the turbulence (disorder) and thus, the rate of internal entropy production is much lower and also exhibits a much less pronounced spatial variation (Annandale, 1987). This is illustrated in Figure 2-10.

As sediment is deposited in the reservoir the velocity of the inflowing water increases, due to a decrease in the channel depth, leading to a more uniform distribution of sediments until a constant value of internal entropy production (P) is reached. This is as illustrated in Figure 2-11. Under these conditions the mean sediment discharge throughout the reservoir approaches a constant value. This results in the longitudinal profile and the rate of internal entropy production being time-independent.

i.e.

$$\frac{dP}{dt}=0$$

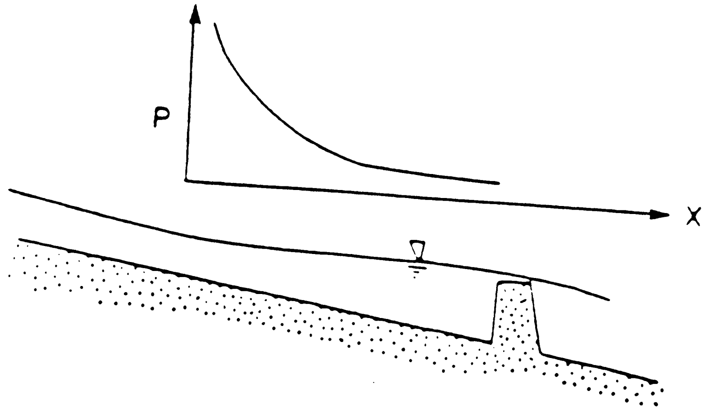

Figure 2-10: Stream power distribution in a reservoir with no deposited sedim

(Annandale, 1987)

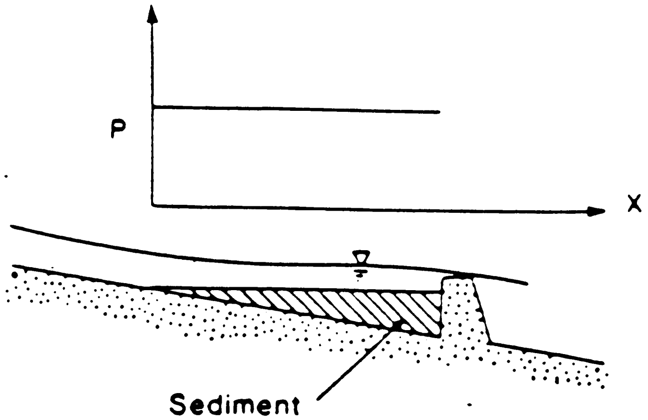

Figure 2-11: Stream power theory in a reservoir with deposited sediment

(Annandale, 1987)

The actual minimum value of the rate of internal entropy production (P) is not an universal constant but varies from case to case and is dependent on the external entropy supply and other limitations imposed on the system (Annandale, 1987). In case of unstable non-equilibrium conditions the shape of the sediment profile changes constantly. Under such conditions, the stream power is minimum when a stable non-equilibrium state is approached asymptotically (Annandale, 1987).

Annandale (1987) applied stream power theory to calculate the reservoir bottom profile for the Glen Alpine reservoir and the Wentzel reservoir in South Africa and as seen in Figure 2-12 and Figure 2-13 the actual and the calculated sediment profiles are in agreement with each other. The broken lines in these figures represent the calculated profiles, whereas the full lines represent the observed sediment profile.

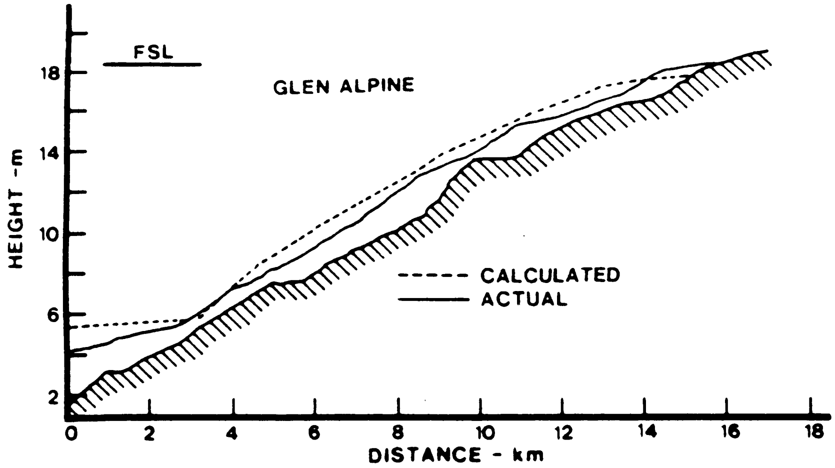

Figure 2-12: Actual and calculated sediment profiles for Glen Alpine Reservoir (Annandale, 1987)

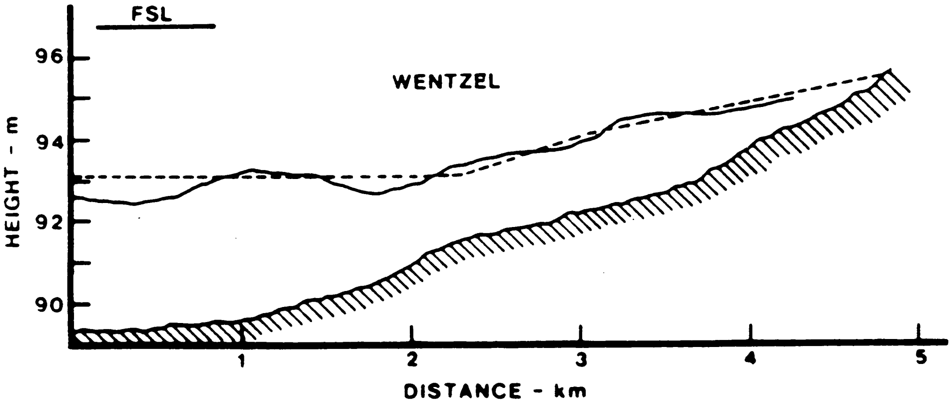

Figure 2-13: Actual and calculated sediment profile for Wentzel Reservoir (Annandale, 1987)

CHAPTER 3: DESCRIPTION OF THE LAKES UNDER STUDY

3.1: Introduction

The purpose of this study is to compare and contrast the varying sedimentation rates and distribution pattern in three selected Iowa lakes. The lakes selected for the study are Pine Lake in Hardin County, Black Hawk Lake in Sac County and Union Grove Lake in Tama County. Pine Lake and Union Grove Lake are river dammed lakes and Black Hawk Lake has a glacial origin. The Pine Lake watershed has two lakes: Lower Pine Lake and Upper Pine Lake. Upper Pine Lake was originally constructed as a silt trap for the Lower Pine Lake. The following sections give a topographic, climatic and geologic description of the three lakes under study. A brief description of the lakes morphology is also included in the following sections.

3.2: Black Hawk Lake

Located on the terminal moraine of the Cary lobe of the Wisconsin glacial surface, Black Hawk Lake is the southernmost glacial lake in Iowa. The lake occupies parts of section 2, 3 and 4 in Viola township and sections 33, 34 and 35 in Wall Lake township of Sac County, Iowa. This lake has had three other names in the past - Boyer Lake, Walled Lake and Wall Lake (Hanson, 1982).

Lake morphology

Black Hawk lake is a very shallow lake. The lake bathymetric map for 1981 indicates that the average depth of the lake was 4.43'. Appendix A shows bathymetric maps for years 1916, 1935, 1973, and 1981. These bathymetric maps were used to perform sedimentation analysis of the lake. Based on the past bathymetric surveys conducted on the lake the morphological characteristics can be summarized as given below in Table 3-1.

Table 3-1: Morphological characteristics of Black Hawk Lake

Year	1916	1935	1973	1981
Area (acre)	799.26	791.12	770.83	763.40
Volume (acre-ft)	3994.01	3349.08	3813.25	3383.22
Average Depth (ft)	4.25	4.23	4.95	4.43
Volume @ (m.s.l.)	1220.5	1220.5	1220.5	1220.5

Geology of the region

The surface geology of the Black Hawk Lake watershed consists primarily of Wisconsin glacial till (71%) and alluvium (18%). About 4% of the watershed is loess capped Kansan glacial till. The remaining portions of the watershed are covered by glacial outwash deposits, gravel pits, marsh, fill and glacial lake sediments (Hanson, 1983). Figure 3-1 shows the surficial geology of Black Hawk lake watershed. Sac

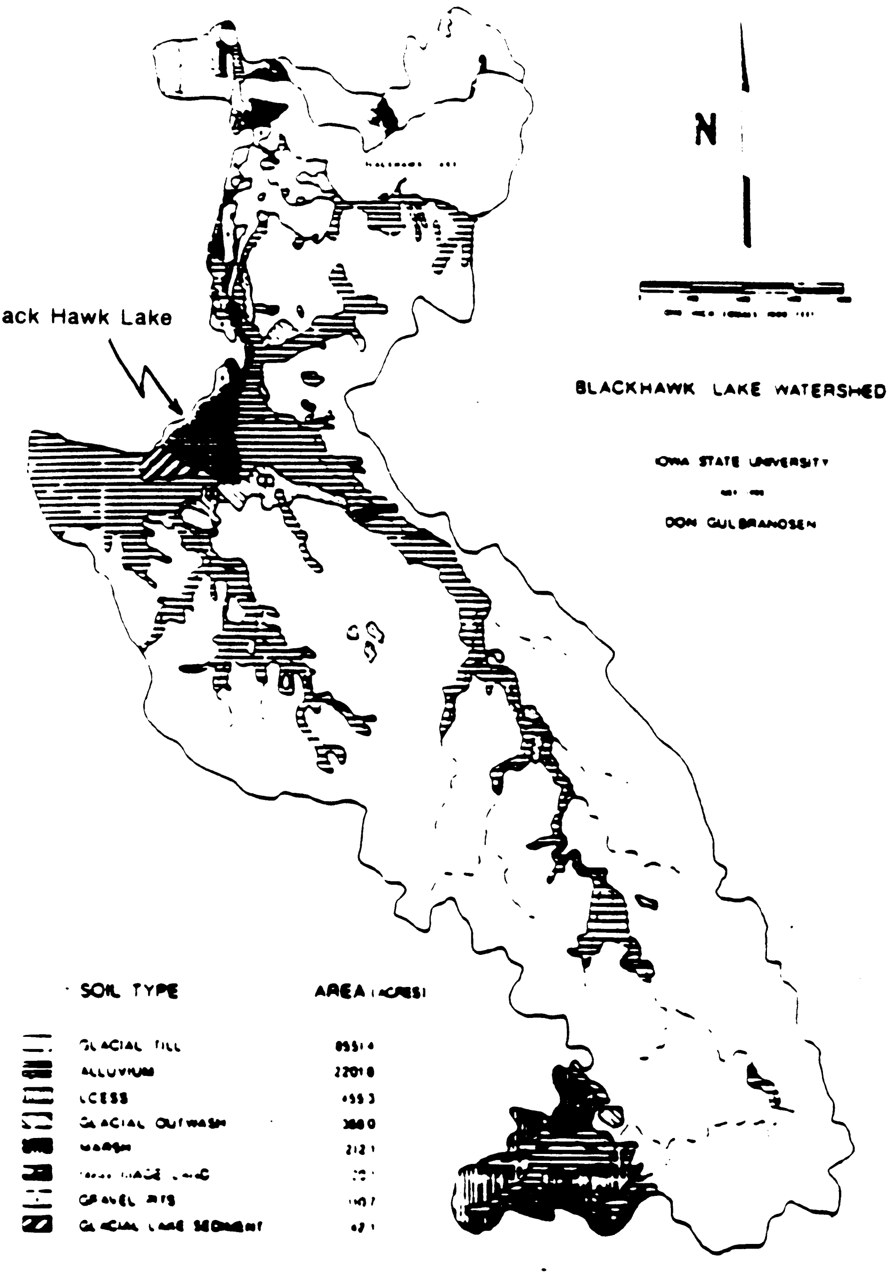

Figure 3-1: Surficial geology of Black Hawk Lake watershed (Hanson, 1982)

County has been subjected to three stages of glaciation. The first two, the Nebraskan and Kansan, covered all of the county. The Tazewell substage of the Wisconsin glacier advanced from the northeast to near the southwest corner of the county, and the Cary substage of the Wisconsin glacier covered approximately the eastern half of the county. Loess ranges from 15' to 20' thick on summits in the southwestern part of the county to about 4' or less near the border of the Cary drift, where Black Hawk lake watershed is situated (SCS, 1979).

Climatic conditions

The annual precipitation in Sac County, Iowa ranges from 28" in the western part of the county to almost 29" in the south-east corner (SCS, 1979). The climate of Sac County, as classified by SCS, is humid to subhumid.

Topography

The soils in the eastern part of the county are nearly level to undulating. In the western part, except the southwest corner of the county, they are mainly gently rolling to hilly. The prevailing slope is to the south and east. Elevation ranges from about 1000' at the low point in the southeastern part of the county to about 1400' near the northwest corner (SCS, 1979).

3.3: Pine Lake

Pine Lake State Park, Hardin County, is about 1/2 mile northeast of Eldora on Iowa Highway 118. The park consists of two interconnected recreational lakes - Lower Pine formed in 1922 by impounding Pine creek near its junction with Iowa River and Upper Pine formed in 1935 by constructing the Upper Pine Lake dam on Pine Creek. Pine lake since its inception has been subjected to sedimentation problems. The eastern portion of the lake is gradually turning into marshlands thereby decreasing the overall depth of the lake and posing potential fish winterkill problems due to small volume of water available to hold the dissolved oxygen in the winter under ice cover (Lohnes et al, 1991).

Lake morphology

An extensive survey of the Upper and Lower Pine lakes was conducted in January 1990 to create the lake bathymetric maps. Lake depth measurements were made by soundings through the ice along 45 transects. A total of 293 soundings were made on the two lakes. Appendix A shows the bathymetric maps of the two lakes based on the surveys conducted. These maps were used to carry out sedimentation analysis of the lake. Based on the previous bathymetric surveys morphological characteristics of Lower Pine Lake can be summarized as in Table 3-2.

Table 3-2: Morphological characteristics of Lower Pine Lake

Year	1922	1932	1950	1990
Area (acre)	69.63	68.44	66.27	63.50
Volume (acre-ft)	680.90	586.35	516.52	354.68
Average Depth (ft)	9.78	8.57	7.79	5.59
Volume @ (m.s.l.)	970.5	971	971	970.5

Geology of the region

As a part of the diagnostic study on Pine Lake to provide the Iowa Department of Natural Resources adequate information for planning a lake restoration program on the lake and its watershed, a surficial geology map as shown in Figure 3-2 was prepared. The surficial geology of the Pine Lake watershed consists mainly of loess (77.9%) and alluvium (17.6%). Sandstone is the parent material of soils which cover 2.8% of the watershed. Glacial till covers 1.3% of the total area and the rest is covered by Eolian sand and water.

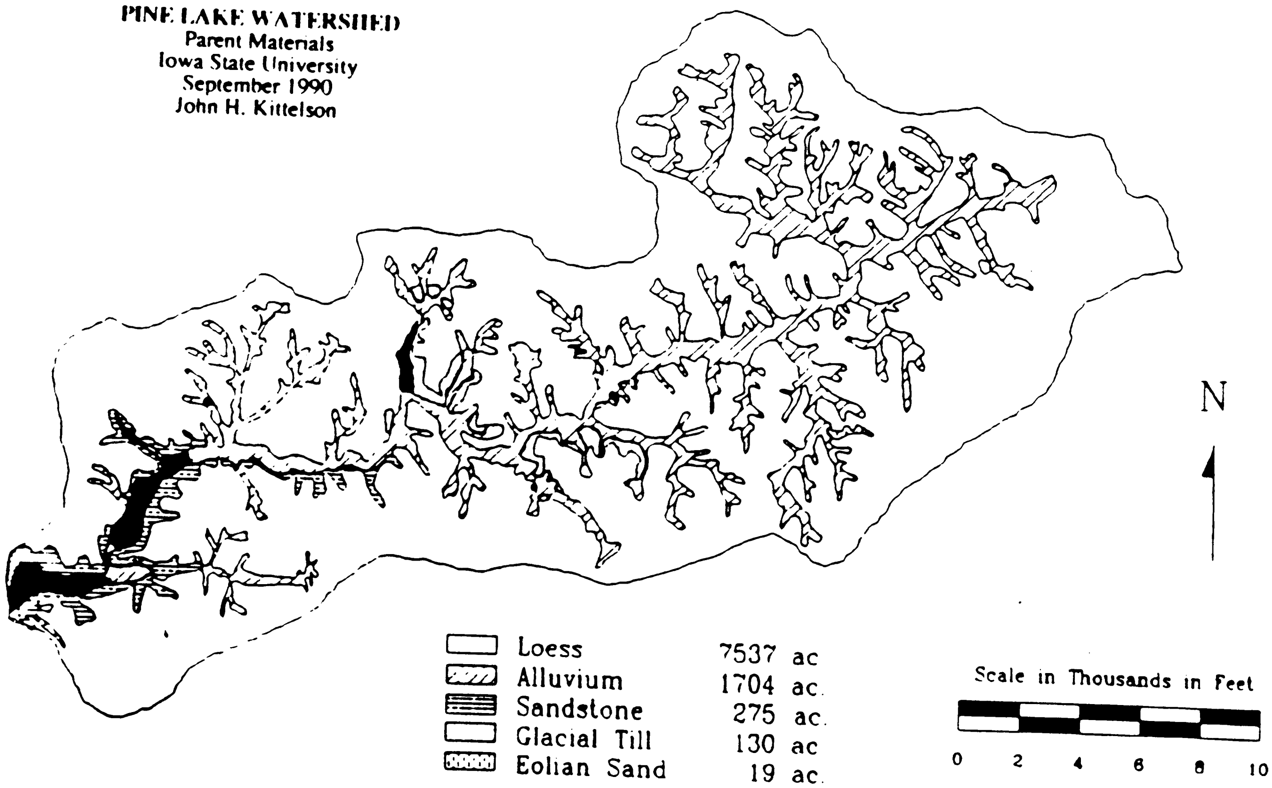

Figure 3-2: Surficial Geology of Pine Lake watershed

Topography

From the SCS soil report for Grundy County and Hardin County as also the USGS topographical maps, it is observed that the topography of the drainage basin is fairly flat to undulated in the upper reaches of the watershed and steeper towards the lake. The elevation varies from 1120' at the eastern portion of the watershed to about 960' near the dam.

Climatic conditions

Approximately 70% of the yearly precipitation of Hardin County and Grundy County, Iowa in which the Pine Lake watershed lies, falls during the period of April to September. The climate in general can be classified as humid to subhumid.

3.4: Union Grove Lake

Union Grove Lake was built in 1934 as an impoundment on Deer Creek which discharges into the Iowa River at Tama, Iowa. The lake lies four miles south of the town of Gladbrook and about fifteen miles northeast of Marshalltown, Iowa.

Geology of the drainage basin

The drainage basin of Union Grove Lake has an area of 6895 acres and is within the region of the Iowa erosion surface of Wisconsin age. This geomorphic surface is characterized by multi-level, stepped erosion surface with both glacial till and loess occupying the uplands (Lohnes et al, 1982). The watershed map of Union Grove Lake is as shown in Figure 3-3. As is seen from the figure, 75% of the watershed is loess and glacial till underlies about 8.5% of the drainage basin and the rest is alluvium, colluvium, terrace deposits and some limestone outcrops.

Lake morphology

Bathymetric surveys were conducted on Union Grove Lake in 1936, 1950 and 1981. Based on these surveys the morphological characteristics of the lake can be summarized as in Table 3-3.

Table 3-3: Morphological characteristics of Union Grove Lake

Year	1936	1950	1950	1970	1981
Area (acre)	118.04	105.3	129.55	116.85	106.62
Volume (acre-ft)	796.62	724.98	998.94	836.12	662.15
Average Depth (ft)	6.75	6.88	7.71	7.16	6.21
Crest Elevation (m.s.l.)	937.6	937.6	939.6	939.6	939.6

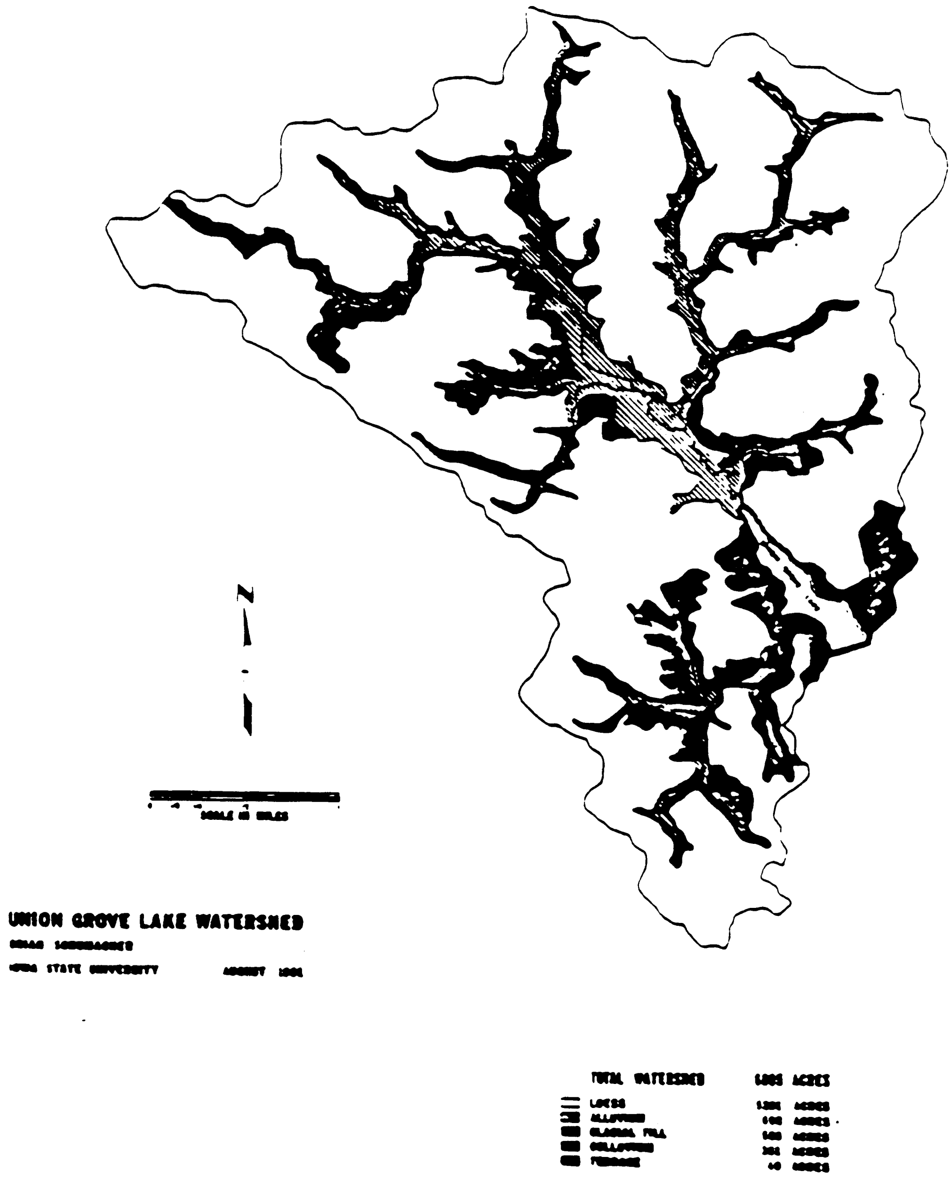

Figure 3-3: Surface geology of Union Grove Lake watershed (Lohnes et al, 1982)

CHAPTER 4: COMPUTER SOFTWARE USED IN THE STUDY

4.1: Introduction

The sedimentation study of the three lakes under consideration involved transferring hand-drawn bathymetric maps into the computer using AUTOCAD and using these data to estimate the amount of sedimentation. This chapter briefly describes the computer software used to estimate the amount of sedimentation and the methods used to make sedimentation computations.

4.2: AUTOCAD

To make the sedimentation computations, the hand-drawn maps were input into the computer using AUTOCAD. AUTOCAD is a computer graphics program with a wide range of applications. Of particular interest to this study is the ability of AUTOCAD to convert the hand-drawn maps into drawing files. This method of transferring a hand-drawn map into a computer perceptible map is referred to as digitizing, and involves converting points along the contours of a bathymetric map into (X,Y,Z) values in Cartesian coordinate system using the TABLET command. AUTOCAD also offers the advantage of obtaining areas bounded by contours of the lake directly, by using the AREA command, thereby avoiding the tedious work associated with the conventional method of planimetering the contour lines to obtain the areas. Figure 4-1 shows a flowchart of the various steps involved in converting a hand-drawn map into computer compatible map using AUTOCAD. The drawing files are then converted by AUTOCAD into DXF (Drawing Interchange file) format, an ASCII file, which can then be used by many third party programs. The DXF files can be converted into data files using ZTAB3, a special utility program developed by the Iowa State University Land Use Analysis Laboratory for translating into ASCII files (of X, Y, Z data) for use by SURFER.

4.3: SURFER

SURFER is a computer software program used as a tool for creating high resolution two and three dimensional graphics. Through various available menus and more than 100 options contour maps and surface plots of XYZ data are created. Figure 4-2 shows a flowchart of the various menus available and their application to this study.

GRID menu

This option converts files of irregularly spaced data points into grid files of regularly spaced data points (nodes) in the XY plane and then interpolates the elevation (Z-coordinate) at each node using various conditions specified by the user. The first step

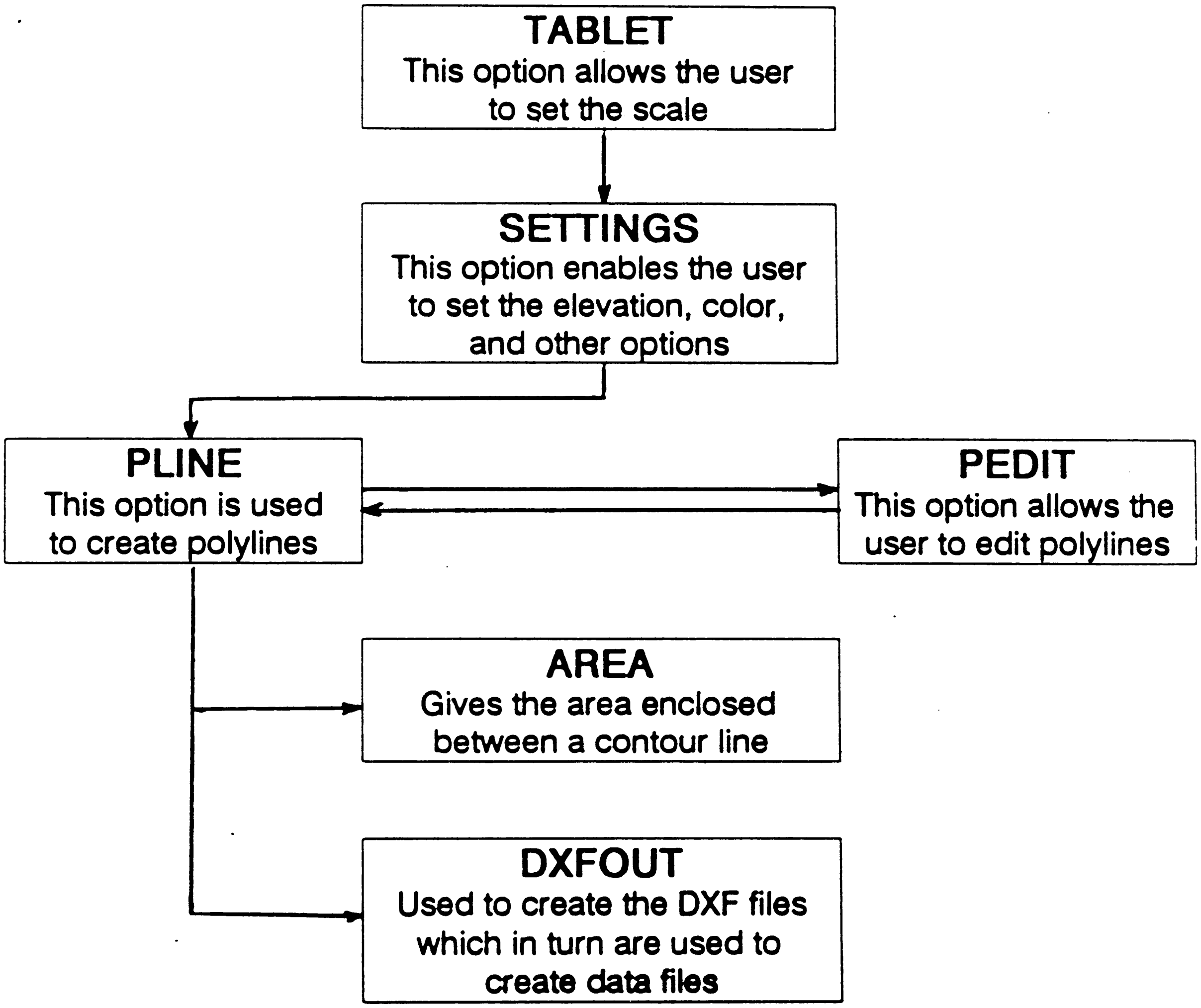

Figure 4-1: Flowchart showing various steps involved in AUTOCAD

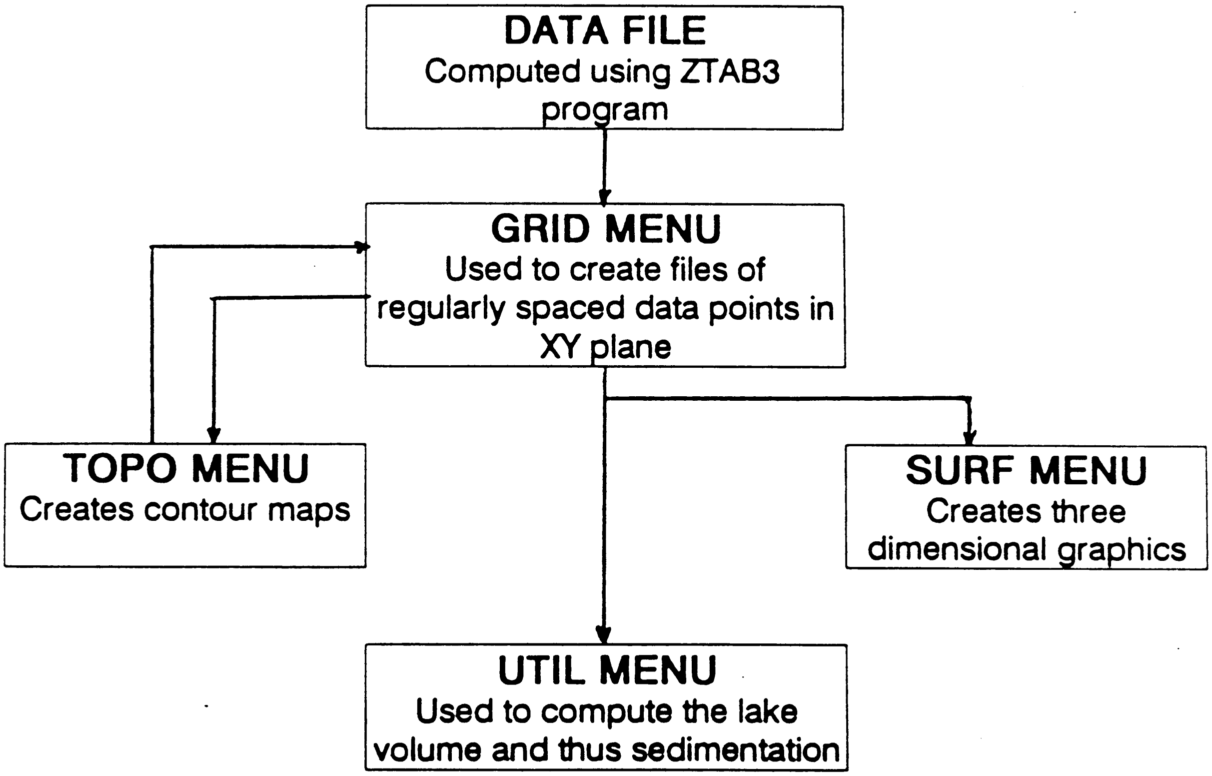

Figure 4-2: Flowchart showing various menus in SURFER

in creating a grid file is to import the XYZ data into the GRID. The GRID file is obtained by specifying different parameters such as grid size, search radius, number of search points and the interpolation method that has to be used.

The grid size is used to specify the density of the final grid. According to Jones (1987) the grid interval should be so chosen that a given grid square contains at the most one data point. A grid size of 50x50 usually gives pleasing results. Maintaining a balance between the accuracy desired and the computer bytes utilized a grid size of 51x51 was used to create the grid files for this study.

There are three gridding methods available in SURFER - Inverse distance, Kriging and Minimum curvature. Inverse distance uses a weighted averaging technique to interpolate grid node elevations from XYZ data. The weights are inversely proportional to the distance from the grid node.

Minimum curvature is the fastest among the three methods. This method first examines all data and sets the nearest grid node to that data value, thereby honoring the data. However this method gives poor results especially when the data are unevenly distributed in space.

Kriging uses geostatistical techniques to calculate the autocorrelation between data points and produce a minimum variance unbiased estimate. The variation is estimated by calculating a semi-varignon, a statistical tool that relates variation to distance (Jones, 1987). In other words, Kriging comes up with a value of elevation at each node based on the elevations of the data points in the user specified search sector. Although in theory

Kriging produces the most accurate estimates, the effectiveness of this method depends on the proper selection of various parameters, such as search radius, search method, number of search points, grid size and search sector (SURFER, manual). This study used Kriging method to make the grid files primarily because the method gave better results.

Search radius and the number of search points tells the computer the area within which it has to scan to make autocorrelation between points while making the grid file. It was observed that the default value of 10 search points and a normal search gave the best results. The search radius has to be set depending upon the size of the lake.

TOPO menu

TOPO is a menu-driven contouring program which makes use of the grid data which may be in binary or ASCII form. Several options are available in the TOPO menu and the user may use the default parameters or specify his/her own. TOPO provides the on-screen contour line editing option which allows alteration of the Z value at any point at a grid node. For nodes near areas of low input data, SURFER correlates and assigns Z value to each node, based on the Z values of the number of user specified points within the search radius. Towards the boundaries, SURFER correlates and draws contours, taking into account the nodes that lie outside the irregularly shaped reservoir.

SURF menu

SURF is an interactive, menu-driven graphic program that creates three dimensional surface representations as a measured perspective drawing on the computer screen. This has potential application in visually analyzing the spatial distribution of sediments in lakes.

VIEW menu

This option enables viewing the contour or the three dimensional plots on the screen before actually plotting them with a plotter or a printer.

UTIL menu

UTIL is a utility program that performs several functions on the XYZ data in the grid files. Of particular importance in this study is the ability of UTIL to calculate the volumes of solids defined by gridded surfaces.

4.4: Sedimentation Computation

Even though several methods are available to estimate the amount of sedimentation, three methods are considered herein. The volume of sediments can be estimated by using the modified prismoidal method; by using SURFER, a computer software program; or by using the universal soil loss equation in combination with the trap efficiency curves. In the first two methods the amount of sedimentation is determined by computing the difference in the lake volume at a fixed level at different time intervals. This level is usually taken as the operating level for the lake. The third method uses empirical relationships which relate the capacity-inflow ratio with the fraction of sediments retained by the lake.

Comparison and differentiation between SURFER and modified prismoidal method is made to evaluate the their applicability for sedimentation computations.

Modified prismoidal method

In this method the lake volume is estimated by considering the slice between any two successive contours as a trapezoid, as illustrated in Figure 4-3. The volume is then computed by averaging the area encompassed by the two contours and then multiplying it by the contour interval. The lake volume at any level can then be estimated by taking the cumulative volume up to that level. The lake bottom does not, however, in reality

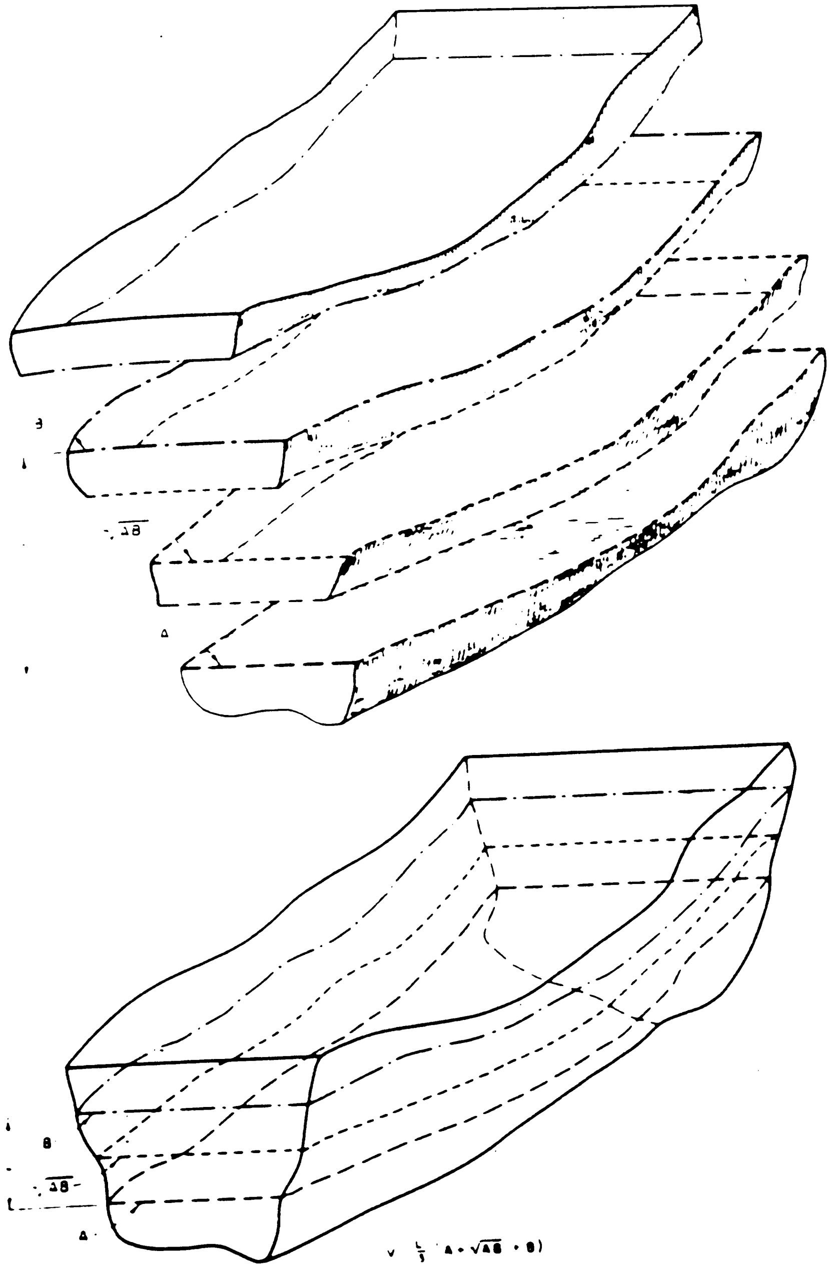

Figure 4-3: Terms of modified prismoidal formula for determining capacity of a reservoir (Source: Vanoni, 1975)

conform to this model, as it is more or less undulated. Nevertheless this model is effective in estimating the lake volume if the contour interval is small as compared to the total depth of the lake.

4.5: Comparison between volume obtained by Modified Prismoidal and SURFER

Comparison was made between these two methods for Union Grove Lake. Figure 4-4 and 4-5 make comparison between the volumes obtained using the modified prismoidal method and SURFER for years 1950 and 1981 respectively. The figures indicate that these methods are more or less comparable with each other. Table 4-1 compares the lake volumes for Union Grove Lake obtained by using SURFER and modified prismoidal method.

Table 4-1: Comparison of lake volume using SURFER and modified prismoidal method

Year	Volume of the lake		Variation (%)
	SURFER	Modified Prismoidal	
1950	998.94	1031.26	3.13
1981	662.15	698.99	5.27

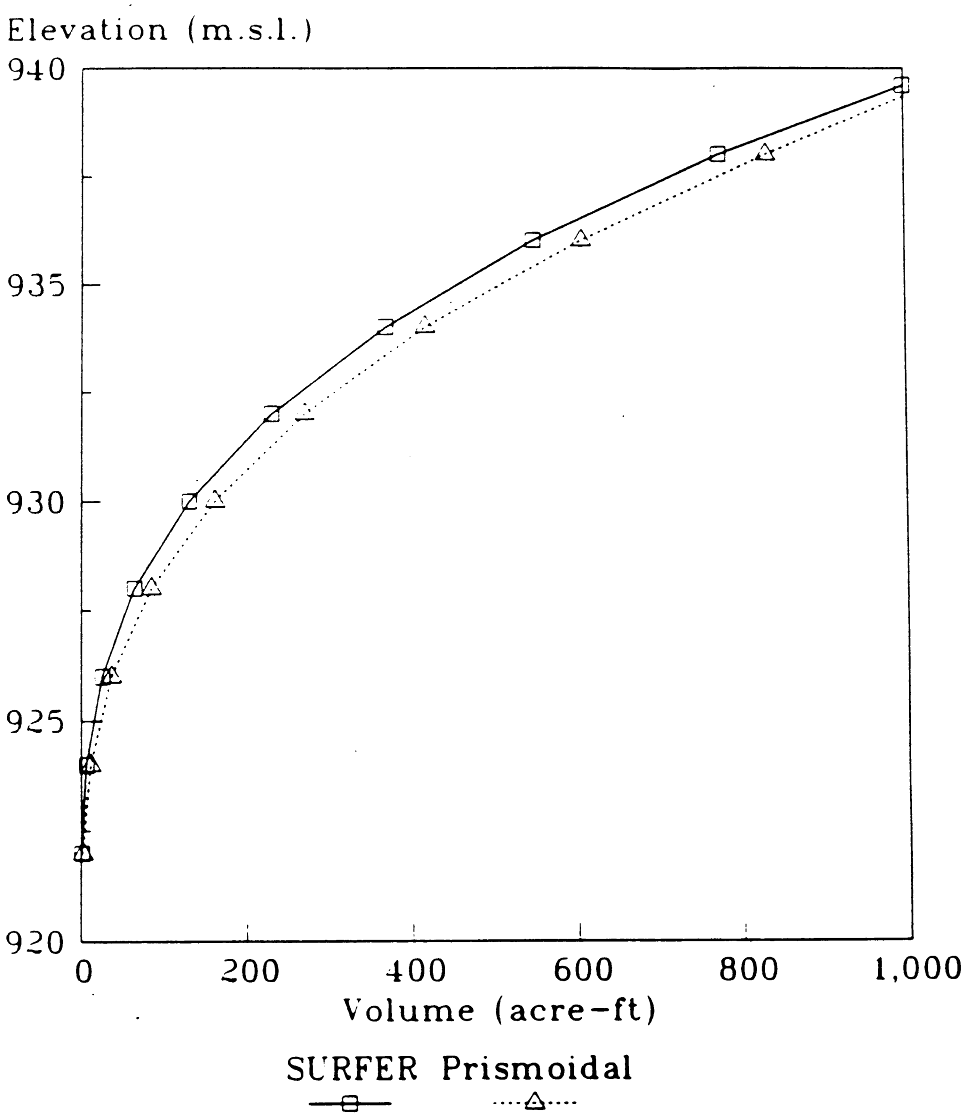

Figure 4-4: Comparison of volume obtained using the modified prismoidal formula and SURFER for Union Grove Lake (1950)

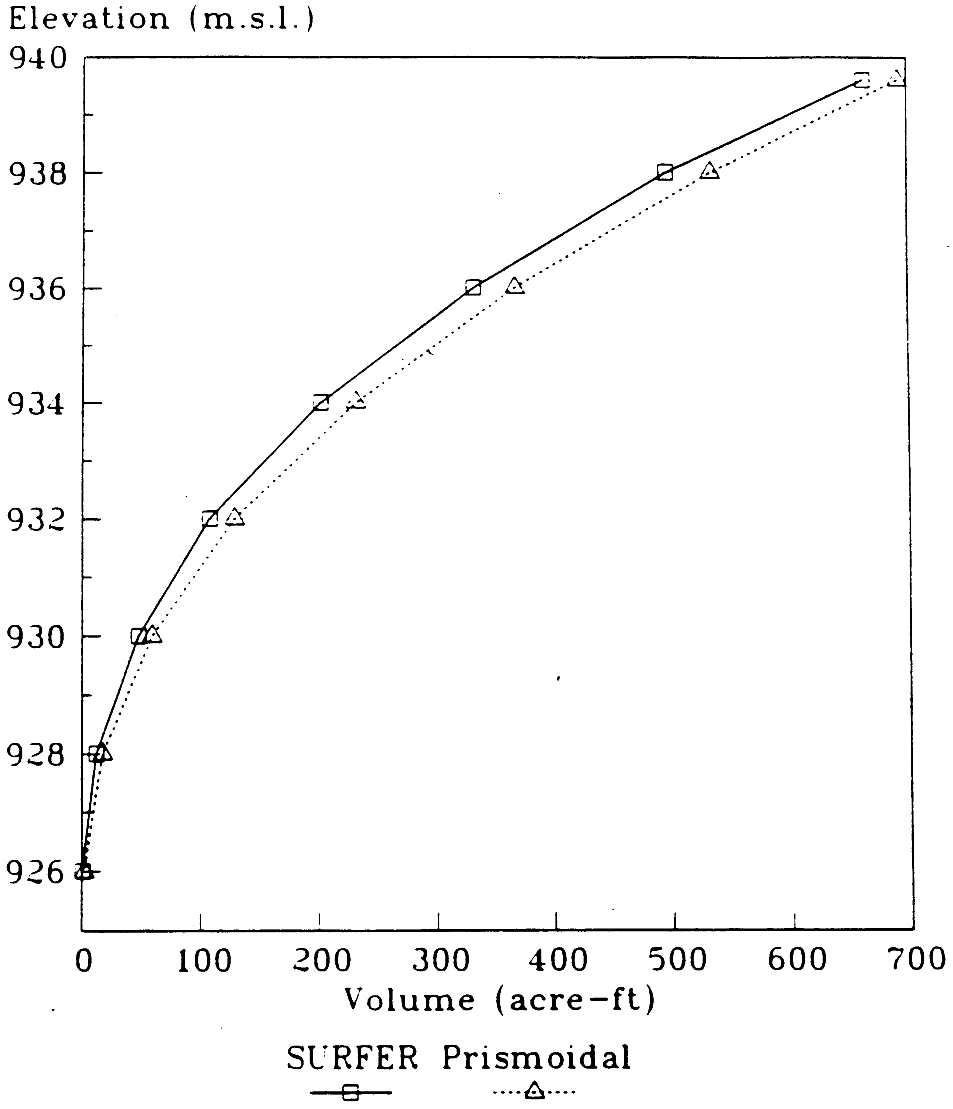

Figure 4-5: Comparison of volume obtained using the modified prismoidal formula and SURFER for Union Grove Lake (1981)

CHAPTER 5: ANALYSIS OF LAKES UNDER STUDY

5.1: Introduction

Sedimentation, as described earlier, is a complex phenomena which depends on numerous factors. The various methods conventionally used to estimate sedimentation have been discussed earlier in this thesis. A novel approach to estimate the amount of sedimentation in reservoirs based on the technique of normalized hypsometric analysis is proposed here.

Researchers have used hypsometric analysis to understand the morphology of a drainage basin. Hypsometric curves, which are essentially elevation vs area curves, were used by Strahler (1952) to describe the horizontal cross-sectional area of a drainage basin to the relative elevation above the basins mouth. Hack (1965) and Moore (1966) have also used the concept of hypsometric analysis to describe drainage basin morphology.

Hypsometric curves can used to describe the morphology of a lake and to estimate the amount of sedimentation. The area-depth hypsometric curves are a useful tool in understanding the spatial distribution of sediments in reservoirs. The shape of a hypsometric curve gives an indication of the spatial distribution of sediments in a reservoir. The amount of sedimentation that has occurred during a time interval is given by the area between the hypsometric curves at two different times.

5.2: Normalization of Hypsometric Curves

Strahler (1952) has suggested the use of hypsometric curves (normalized elevation vs normalized area) to compare drainage basins irrespective of their sizes. Normalization is the representation of two variables as a ratio considering the maximum value in the range as unity; so that the other values in the range will vary from 0 to 1 (0% to 100%).

Sedimentation involves a decrease in the lake capacity due to the deposition of the suspended sediments over the lake bottom. As a part of this research, the elevation and volume of the lakes were plotted on normalized basis. The normalized elevation was taken as the y-variable and the normalized volume was taken as the x-variable.

5.3: Sedimentation analysis of lakes under study

SURFER was applied for the sedimentation analysis of the three lakes under study. The analysis included comparing reservoir elevation with reservoir volumes to estimate the amount of sedimentation and reservoir elevation with reservoir area to characterize the morphology of the lakes. Three types of comparisons were made for the lakes:

- Reservoir elevation vs reservoir area
- Reservoir elevation vs reservoir volume
- Normalized reservoir elevation vs normalized reservoir volume

Hack (1965) and Moore (1966), used normalization technique to compare drainage basins. In order to minimize the bias induced by localized abnormalities, instead of normalizing to the maximum depth, normalization was done at an arbitrary value of area. Similarly in this study, to partially eliminate the bias due to localized abnormalities, normalization of the reservoir elevation was done approximately at the 5% lake area contour.

5.4: Lakes under study

Lower Pine Lake

Lake bathymetric surveys were conducted on the Lower Pine Lake in 1925, 1932 and 1950. In January 1990, an extensive survey of the lake bottom was carried out. The elevation and volume of the lake were compared for these four years for which data were available. A graphical representation of these data is as shown in Figure 5-1. Active sedimentation of the reservoir is apparent from the decreasing volume of the lake. It is interesting to notice that in the case of this lake the rate of sedimentation is high during the earlier years and then decreases. This can be attributed to the following reasons:

- Construction of the Upper Pine Lake which serves as a "silt trap" for the Lower Pine, thereby trapping the eroded soils of the watershed.
- Decrease in trap efficiency of the reservoir due to decreased volume.

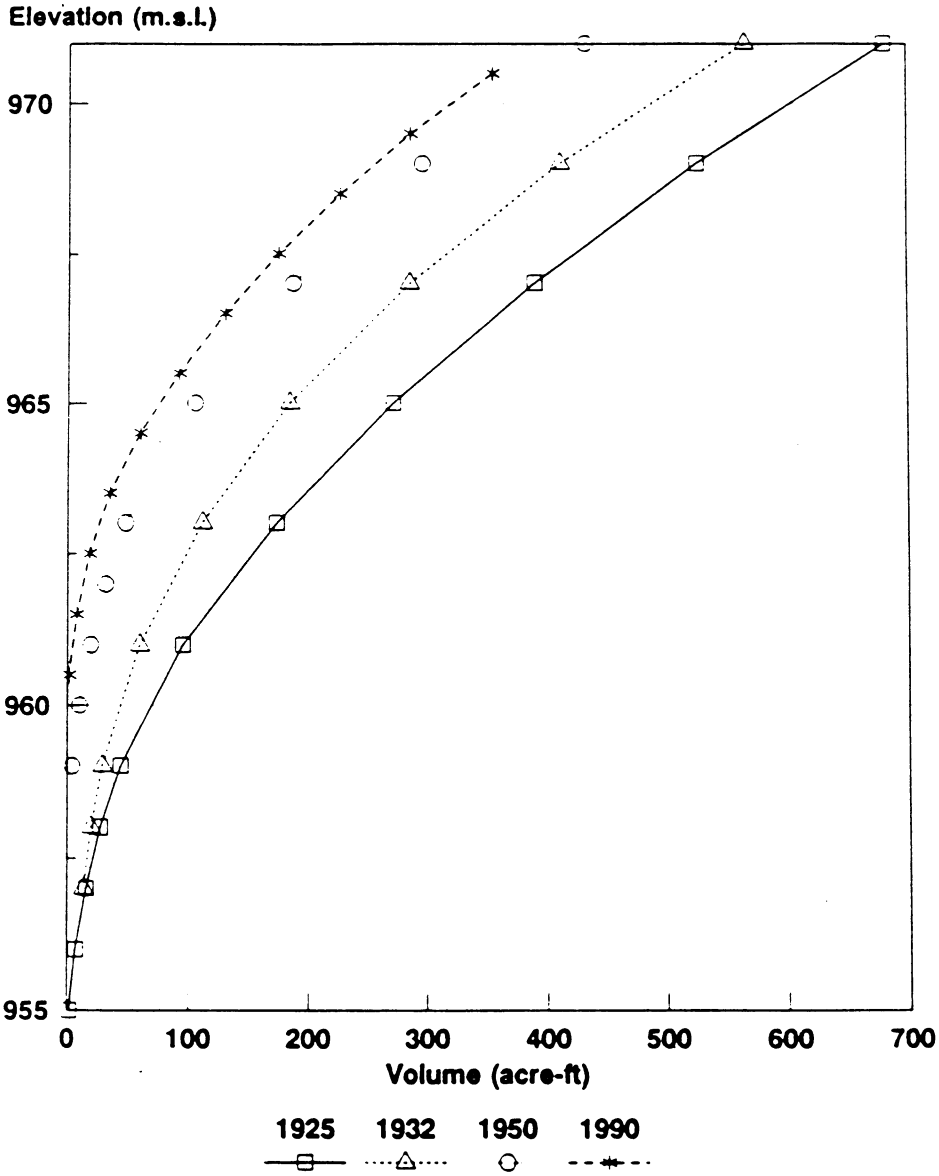

Figure 5-1: Elevation vs volume (Lower Pine Lake)

In order to describe the morphology of the lake the elevation of the lake was compared with the lake area and the graphical representation of the comparison is given in Figure 5-2. The lake had a maximum depth of 19.6' and an area of 70.5 acres when it was constructed. Later on sediments were gradually deposited and the lake decreased in area due to delta formation. The depth of the lake also decreased considerably and the maximum depth of the lake was 14.8' in 1990.

Table 5-1 gives the sedimentation analysis of Lower Pine Lake.

Table 5-1: Sedimentation analysis of Lower Pine Lake

Year	lake volume (acre-ft)	Total sedimentation (acre-ft)	sedimentation rate (acre-ft/year)
1922	680.9		
		130.6	13.06
1932	550.3		
		194.9	3.57
1950	486.0		
		326.2	3.28
1990	354.7		

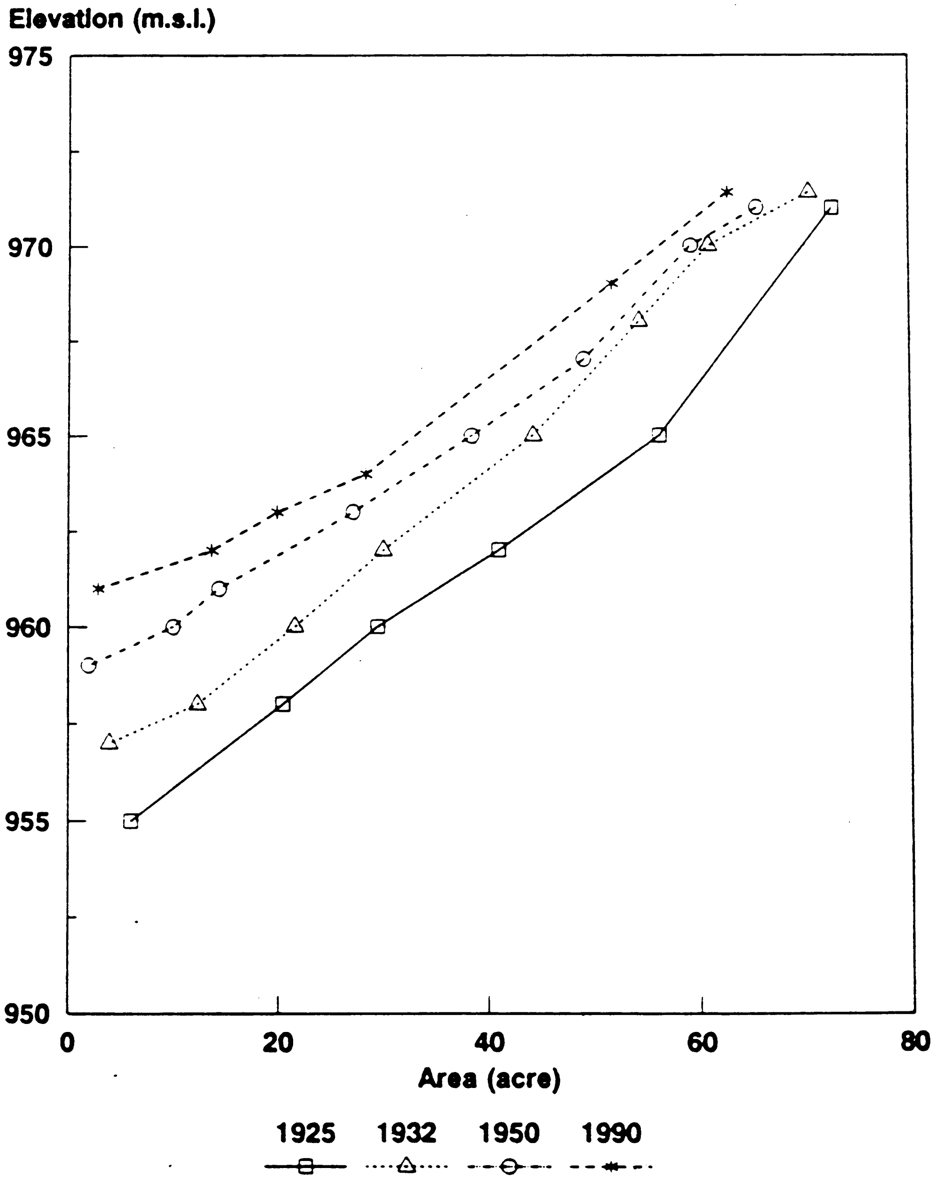

Figure 5-2: Elevation vs area (Lower Pine Lake)

Normalization was applied to Lower Pine lake by comparing normalized elevation with normalized volume. Figure 5-3 shows the elevation and volume relationship on a normalized basis. It is seen from the figure that the normalized curves for the four years, for which the data were available, almost overlap. This leads to the conclusion that this dammed lake has a typical characteristic curve, or a "signature", associated with it when compared on a normalized volumetric basis.

Union Grove Lake

Before proceeding with the sedimentation analysis of Union Grove lake it is essential to recognize two things pertaining to the lake:

- The dam crest was raised by 2' in 1954 thereby increasing the reservoir capacity.
- The lake was dredged in the 1970s and no data were found regarding the volume of sediments removed from the reservoir.

Bathymetric surveys were conducted on Union Grove Lake in 1936, 1950, 1970 and 1981. The plot for elevation vs volume for the lake is as shown in Figure 5-4. From Table 5-2, it is seen that the sedimentation rate in the earlier stages of the reservoir was 5.12 acre-ft/year. After raising the dam crest the average sedimentation increased and was 8.14 acre-ft/year. This probably is due to altering the natural equilibrium of the watershed due to raising of the dam crest.

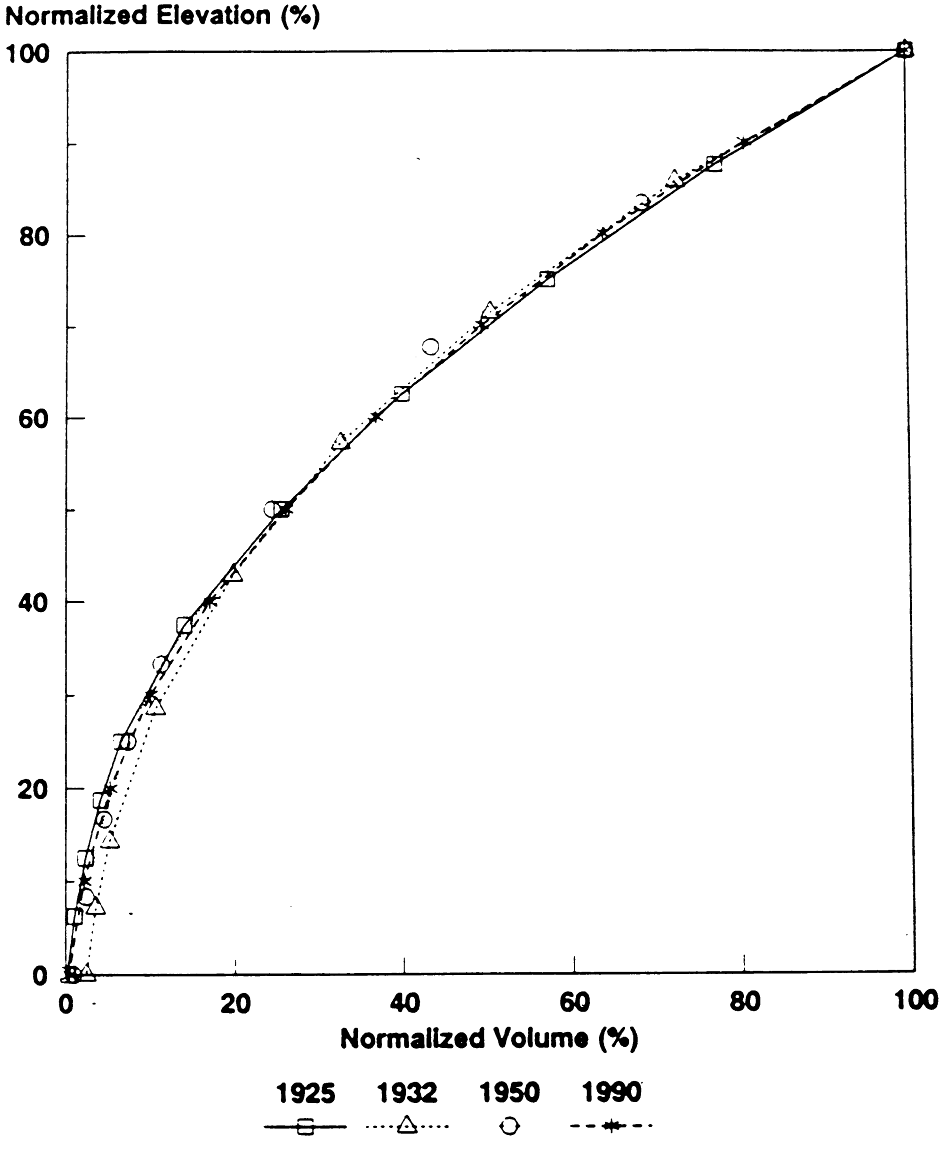

Figure 5-3: Normalized elevation vs normalized volume (Lower Pine Lake)

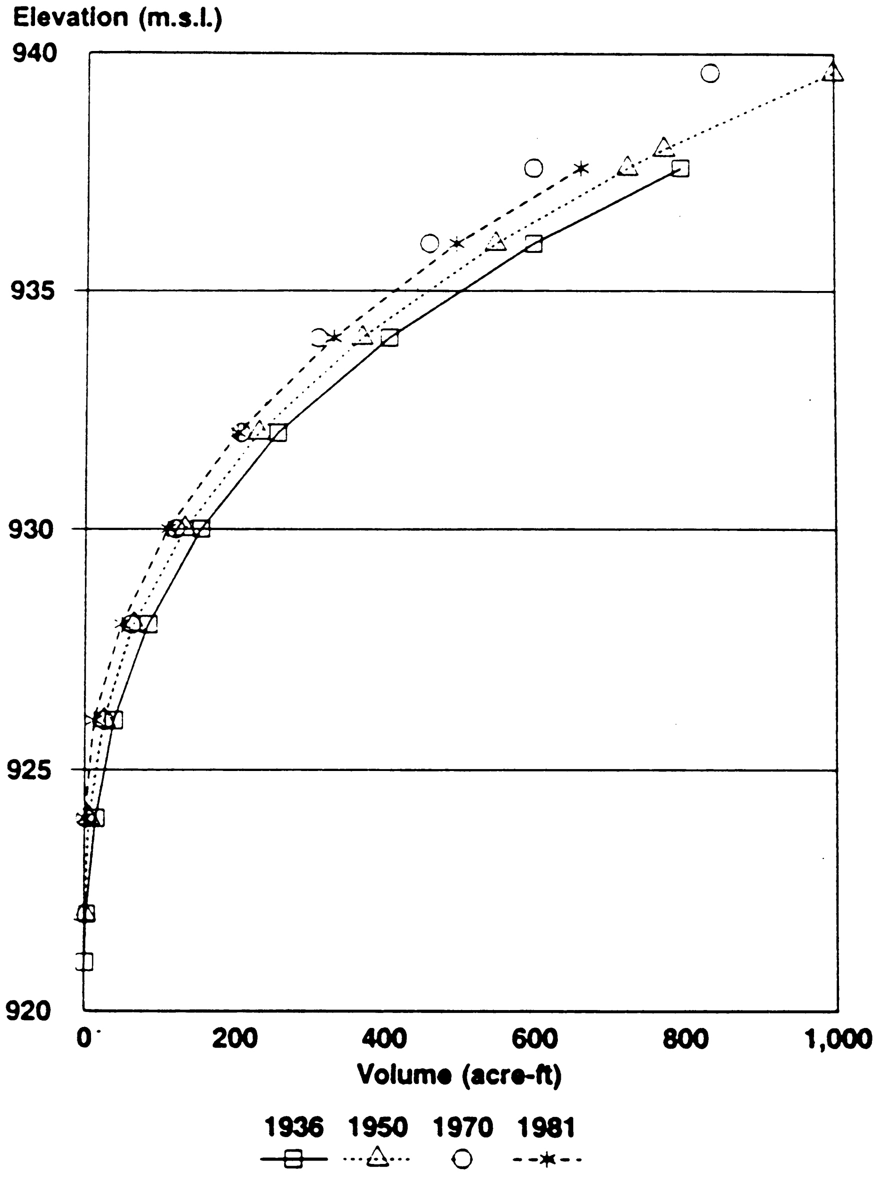

Figure 5-4: Elevation vs volume (Union Grove Lake)

Table 5-2: Sedimentation analysis for Union Grove Lake

Year	Volume of lake (acre-ft)	Volume @ elevation (ft)	Sedimentation (acre-ft)	Average sedimentation (acre-ft/year)
1936 1950	796.62 724.98	937.6 937.6	71.64	5.12
1950 1970	998.94 836.12	939.6 939.6	162.82	8.14
1970 1981	836.12 662.15	939.6 939.6	???	???

Area-elevation relationship was studied to account for the morphological characteristics of the reservoir. As seen from Figure 5-5, the area of the lake has decreased over the years due to delta formation. The area of the lake in 1981 is greater than it was in 1950. This is explained by the fact that the crest of the dam was raised by 2' in 1954 to increase the reservoir capacity. The maximum depth of the lake was 17.8' when it was formed in 1936 and it decreased to 14.2' in 1981 due to deposition of sediments.

Normalization was applied to Union Grove Lake by comparing the normalized elevation with the normalized volume. Figure 5-6 shows the elevation and volume relationship for Union Grove Lake on a normalized basis. Again as was the case with Lower Pine Lake the normalized curves for the years for which the data were available nearly overlap.

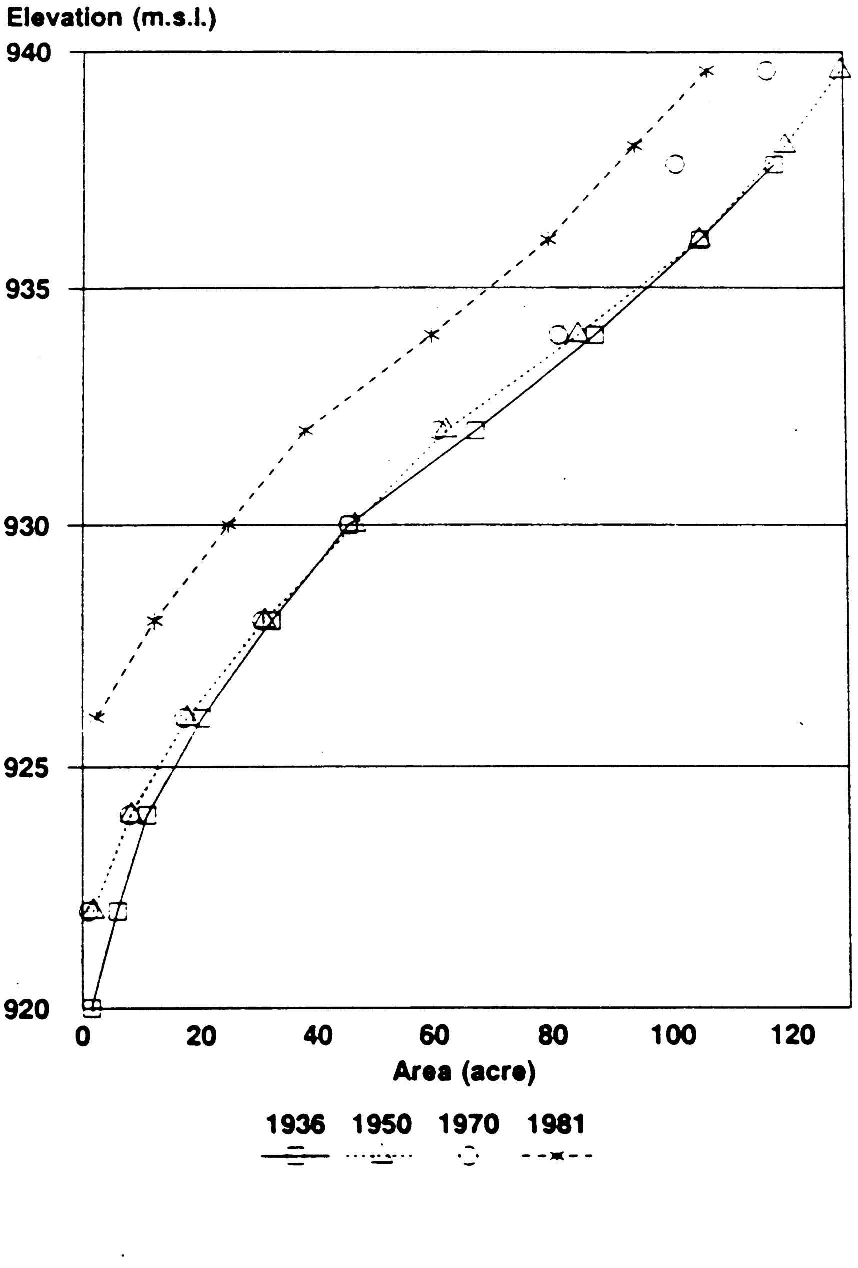

Figure 5-5: Elevation vs area (Union Grove Lake)

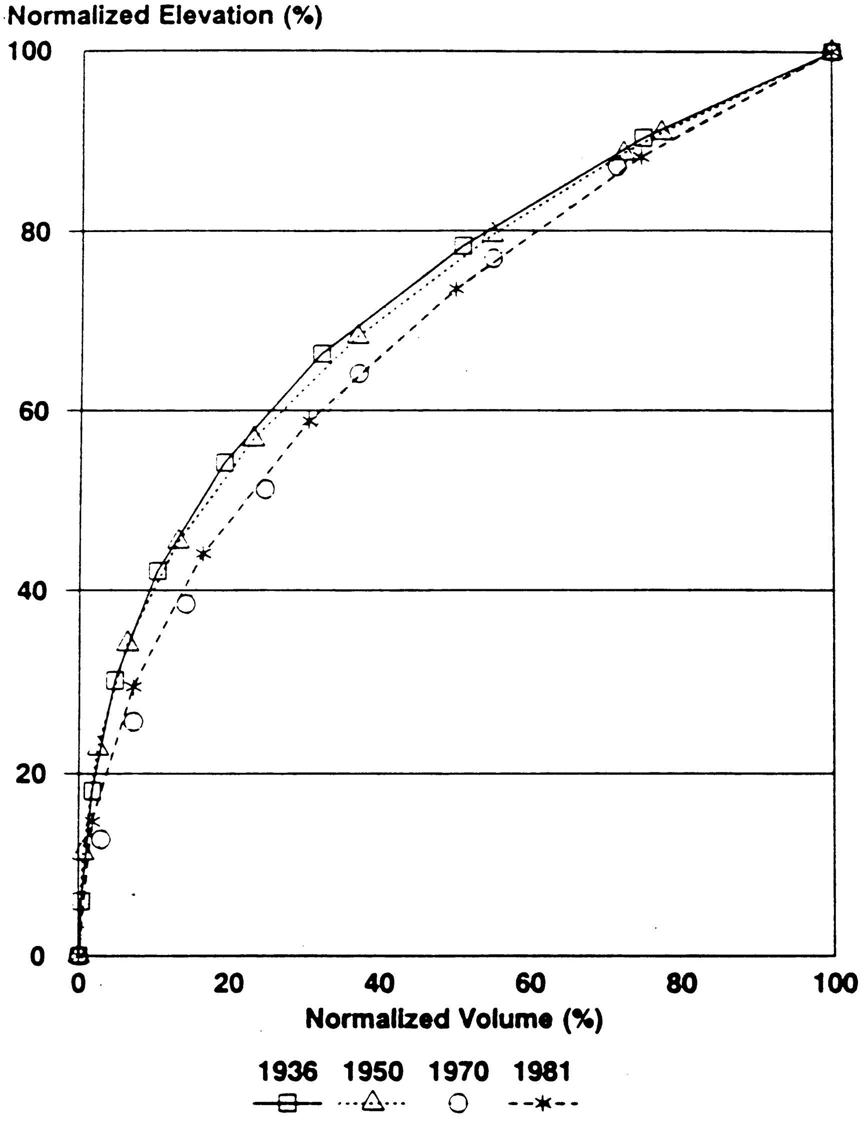

Figure 5-6: Normalized elevation vs normalized volume (Union Grove Lake)

When a lake is dredged or the dam crest is raised, the resulting normalized elevation vs normalized volume curve is somewhat different from the characteristic curve for the lake. It should be noted that the normalized curve for 1981 nearly overlaps the curves for 1970. Also, as seen from the figure the curve for 1981 is in between the curves for 1970 and 1950, thereby suggesting that

- the lake is gradually attaining its characteristic curve prior to dredging of the reservoir and raising of the dam crest, or
- the lake has attained a new equilibrium

From the stream power theory it follows that the sediments are deposited in such a way that the rate of internal entropy production is minimized. When a lake is dredged, the natural equilibrium is altered, disorder increases, and therefore the rate of internal entropy production increases. Sediments are then deposited in such a way that equilibrium is attained. Through a complicated process which is influenced by numerous interrelated factors like fluctuating water levels, temporal variations in flow and sediment discharge, density currents etc., the lake tends to attain its original characteristic curve.

Black Hawk Lake

Before describing the sedimentation analysis of Black Hawk Lake it is essential to note that the lake was dredged in 1938-39 and no documentation of the volume of dredged sediments was found. Bathymetric surveys were conducted on Black Hawk Lake in 1916, 1935, 1973, and 1981, and the data obtained from these surveys were used in conducting the sedimentation analysis.

The graphical representation of the elevation and volume relationship for Black Hawk Lake is given in Figure 5-7. As seen from the figure the lake has decreased in volume over the years due to sedimentation. To describe the morphological characteristics of the lake a graphical representation of elevation vs area, as shown in Figure 5-8, was used. It is seen that the area of the lake has decreased over the years due to sedimentation of this glacial lake. Table 5-3 gives the sedimentation analysis of Black Hawk Lake. As seen from the table, the initial rate of sedimentation was 33.94 acre-ft/year. When the lake was dredged in 1973, the natural equilibrium of the system was altered and this resulted in increased sedimentation. The sedimentation rate after dredging was found out to be 53.75 acre-ft/year.

Table 5-3: Sedimentation analysis of Black Hawk Lake

Year	Volume of lake (acre-ft)	Volume @ elevation (ft)	Sedimentation (acre-ft)	Average sedimentation (acre-ft/year)
1916	3994.01	1220.5	644.93	33.94
1935	3349.08	1220.5		
1973	3813.25	1220.5	430.03	53.75
1981	3383.22	1220.5		

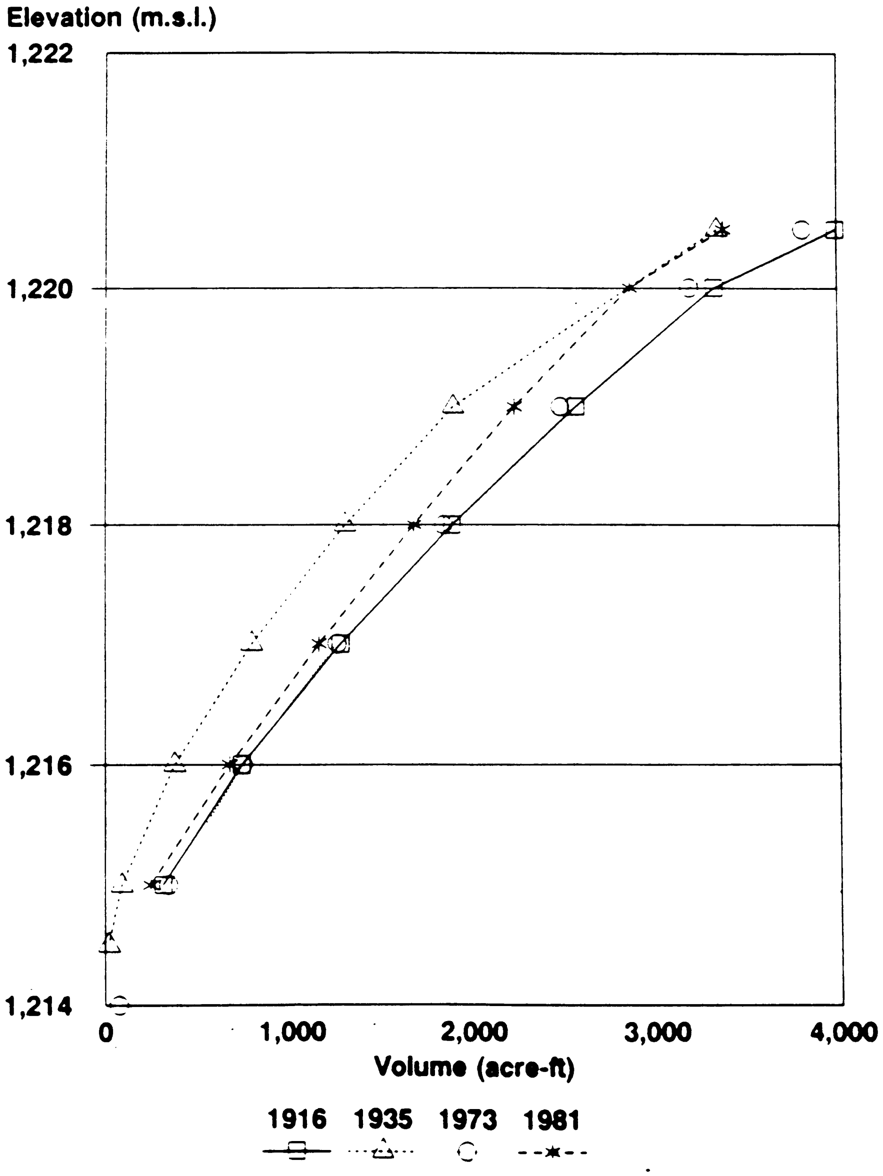

Figure 5-7: Elevation vs volume (Black Hawk Lake)

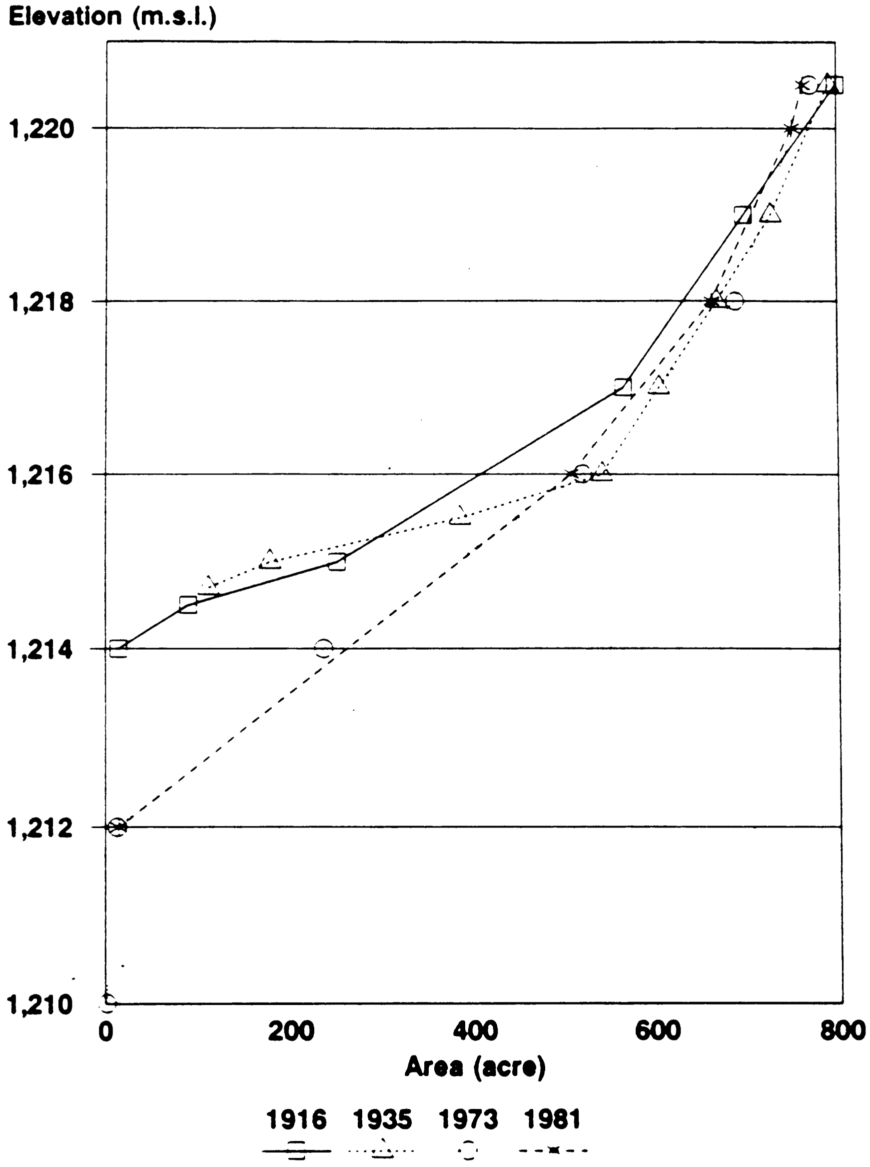

Figure 5-8: Elevation vs area (Black Hawk Lake)

Normalization was applied to Black Hawk Lake. Again, as seen in Figure 5-9, as in the case of the other two lakes the normalized curves almost overlap. However the normalized curve for Black Hawk lake was considerably straight as compared to the other two lakes. As seen from the figure the NENV curve for 1973 and 1981 is slightly differs from the previous two dates. This can be attributed to the dredging of the lake. However it is also seen from the figure that the lake is gradually attaining its original characteristic curve.

Figure 5-10 shows the three lakes on a normalized basis on the same graph. It is seen from the figure that the NENV curves for the three lakes under study are quite different from each other. These varying shaped curves are a reflection of the various watershed characteristics, the shape of the reservoir itself, the reservoir bottom profile, and the reservoir side slopes, which is characteristic of each drainage basin.

5.5 Factors affecting NENV curve

The shape of normalized elevation and normalized volume (NENV) curve depends upon various interrelated factors which include:

- Shape of the reservoir
- Sideslopes of the reservoir
- Spacing of contour lines
- Distribution of relief in the reservoir

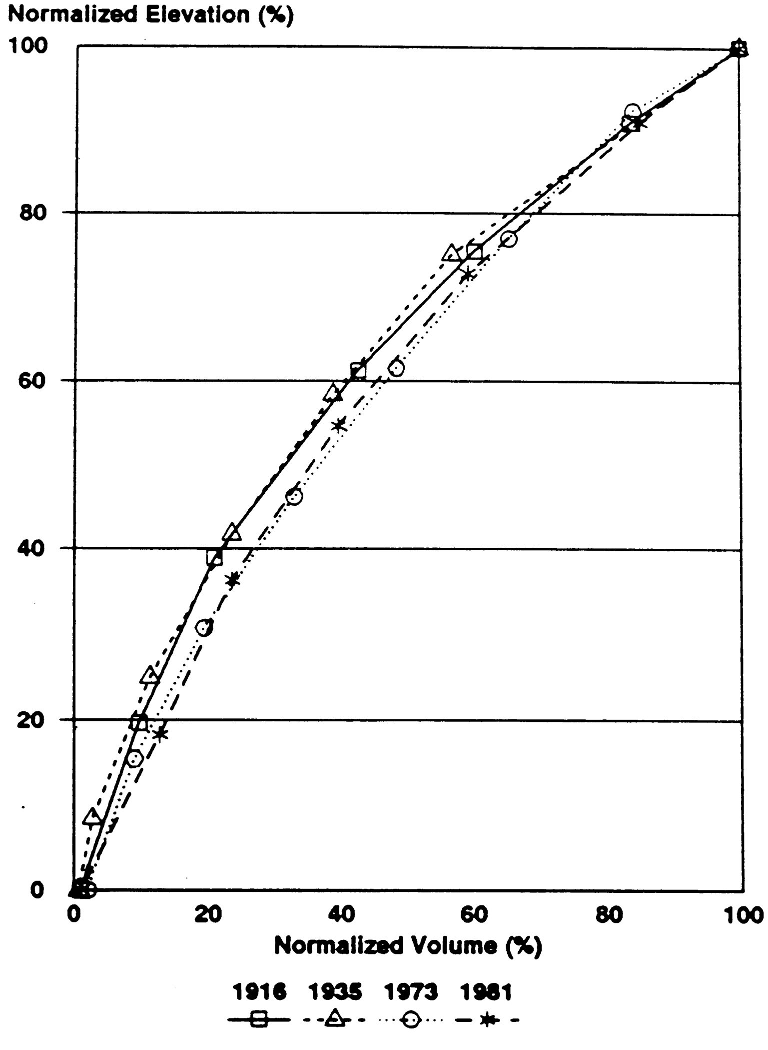

Figure 5-9: Normalized elevation vs normalized volume (Black Hawk Lake)

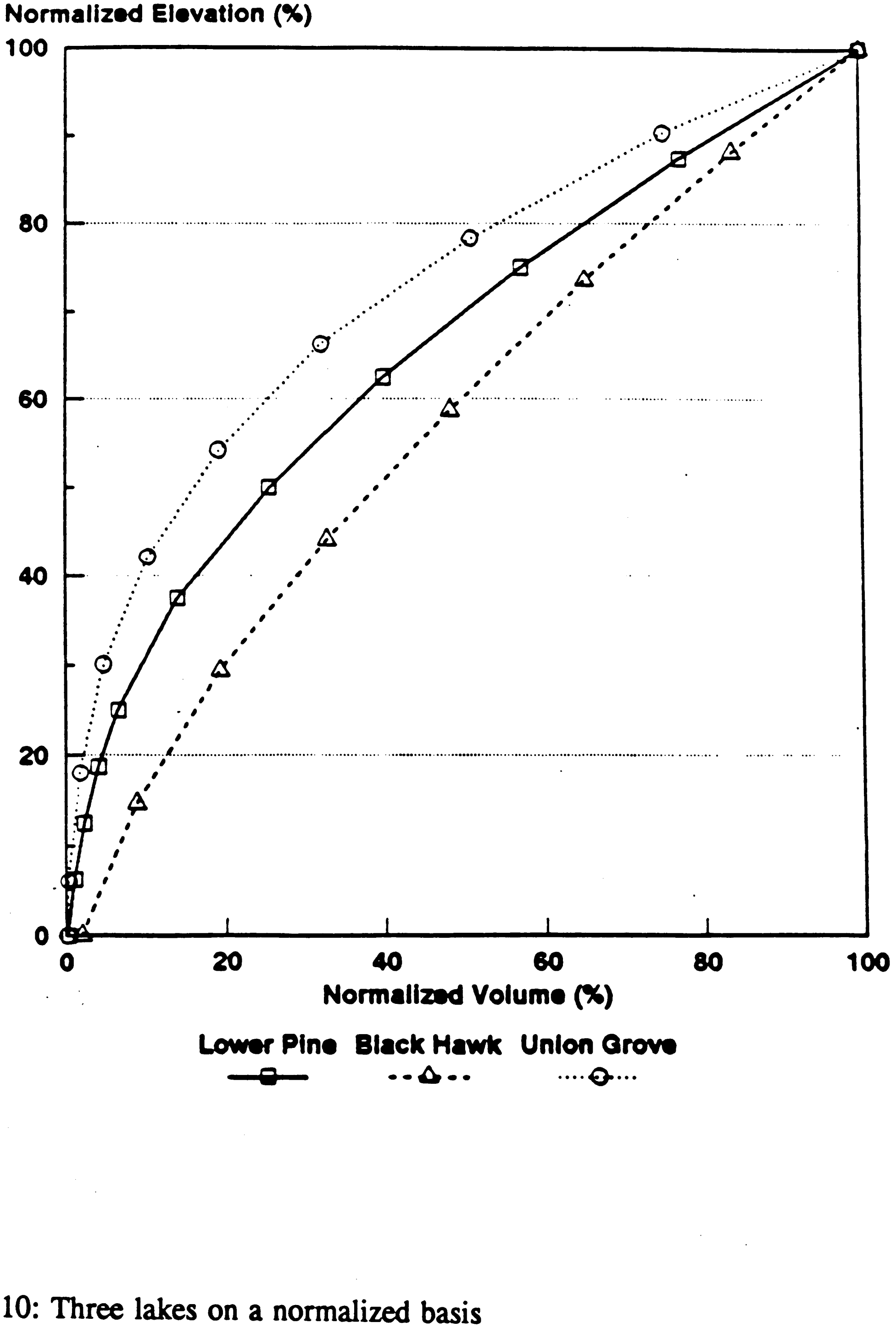

Figure 5-10: Three lakes on a normalized basis

Studies conducted as a part of this research on the three lakes under study revealed that the NENV curves for a particular lake are time invariant. This means that the NENV curves for the lake remain approximately the same over time. This is in agreement with the concept of time independence and dynamic equilibrium for reservoirs as explained by the stream power theory.

In order to illustrate the effect of the shape of the reservoir, reservoir side slope, and the spacing of contour lines (which depends on the distribution of relief in the reservoir) on the NENV curves, the curves were plotted for various three dimensional figures.

Shape of the reservoir

To illustrate the effect of the shape of the reservoir on the normalized curves, five simple three dimensional figures of varying shapes were selected. The three dimensional figures chosen for the geometrical exercise were a hemispherical bowl, a paraboloid, a cone, a cylinder, and a rectangloid. The NENV curves for these five figures are as shown in Figure 5-11. As seen from the figure, the curve for the hemisphere is the steepest, and that for the rectangloid and the cone is straight. In the case of a rectangoloid and a cylinder, the sides are vertical and so the area at any horizontal cross-section remains the same. Therefore, the normalized volume increases linearly with the normalized elevation. The curve for a paraboloid is steeper than that of a cone. The

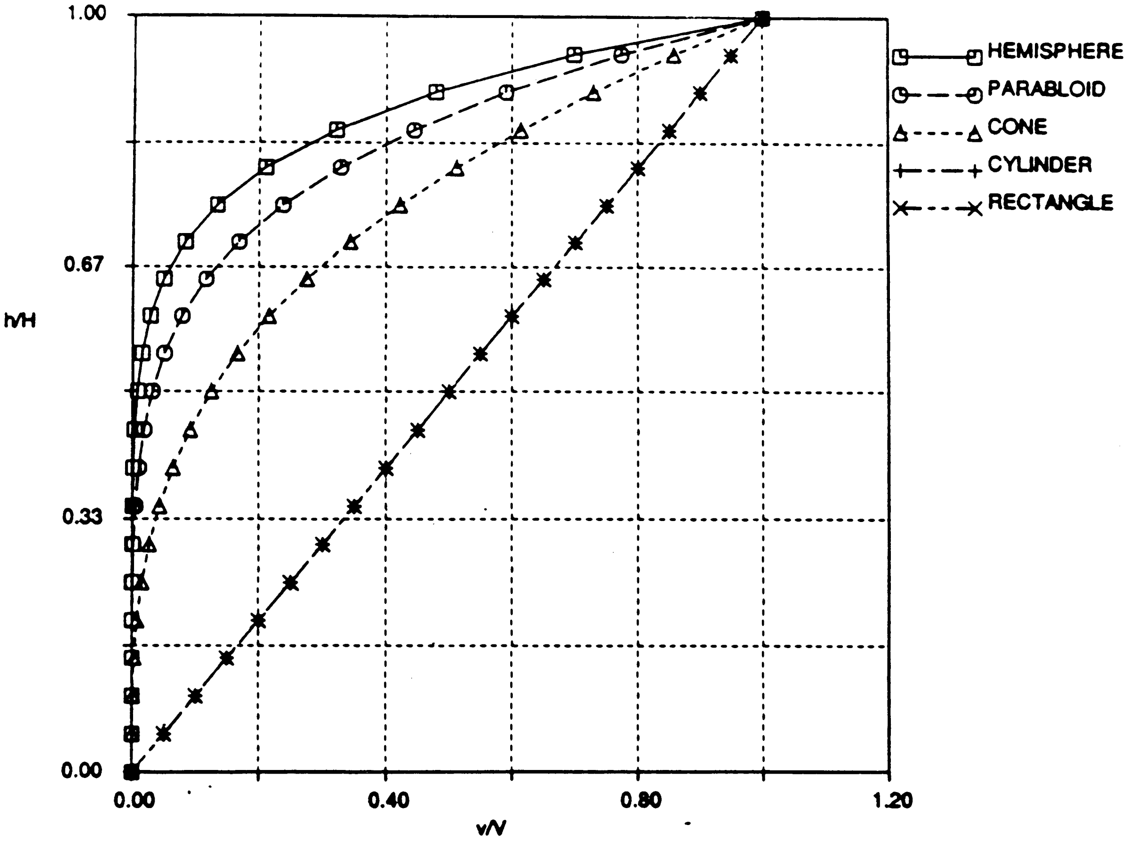

Figure 5-11: Effect of shape of reservoir on NENV curves

varying shapes of the simple three dimensional figures contribute to the variability of the curves, thereby making it evident that the shape of the reservoir is an influencing factor for the shape of NENV curves.

Reservoir side slope

The effect of reservoir side slope on the curves was studied by plotting the NENV curves for five simple three dimensional figures as shown in Figure 5-12. These curves, as shown in Figure 5-13, show that the curve for fig.4 is the steepest and that for a rectangloid is a straight line. The curves suggest that the reservoir sideslope influences the shape of the NENV curves.

Spacing of contour lines

The manner in which the contour lines are spaced depends on the distribution of relief in the reservoir, and the slope of the reservoir bed. To illustrate the influence of this factor on the NENV curve, two hypothetical reservoir shapes as shown in Figure 5-14 were selected. The curves for the two hypothetical reservoirs is as shown in Figure 5-15. As seen from the plot the curves are conspicuously different for the middle portion where the spacing of contour lines is different.

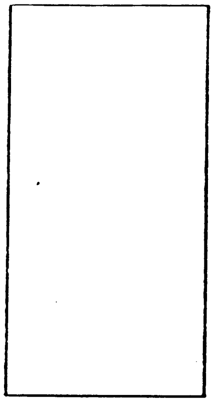

FIG. 1

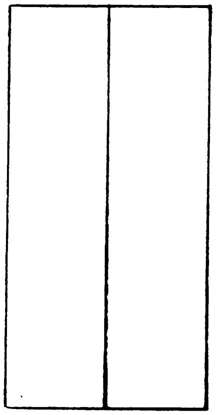

FIG. 2

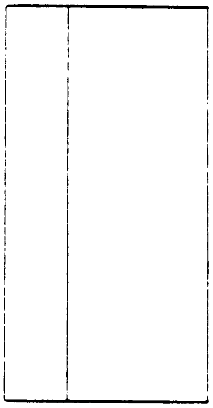

FIG.3

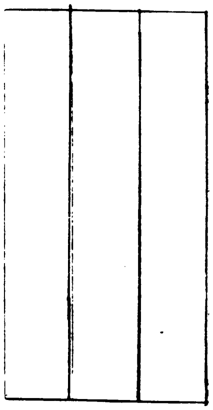

FIG.4

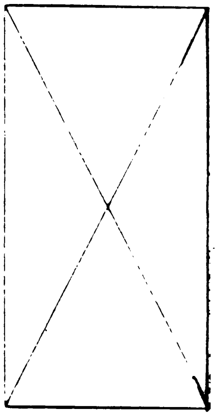

FIG.5

Figure 5-12: Hypothetical shapes chosen to illustrate effect of side slopes on NENV curves

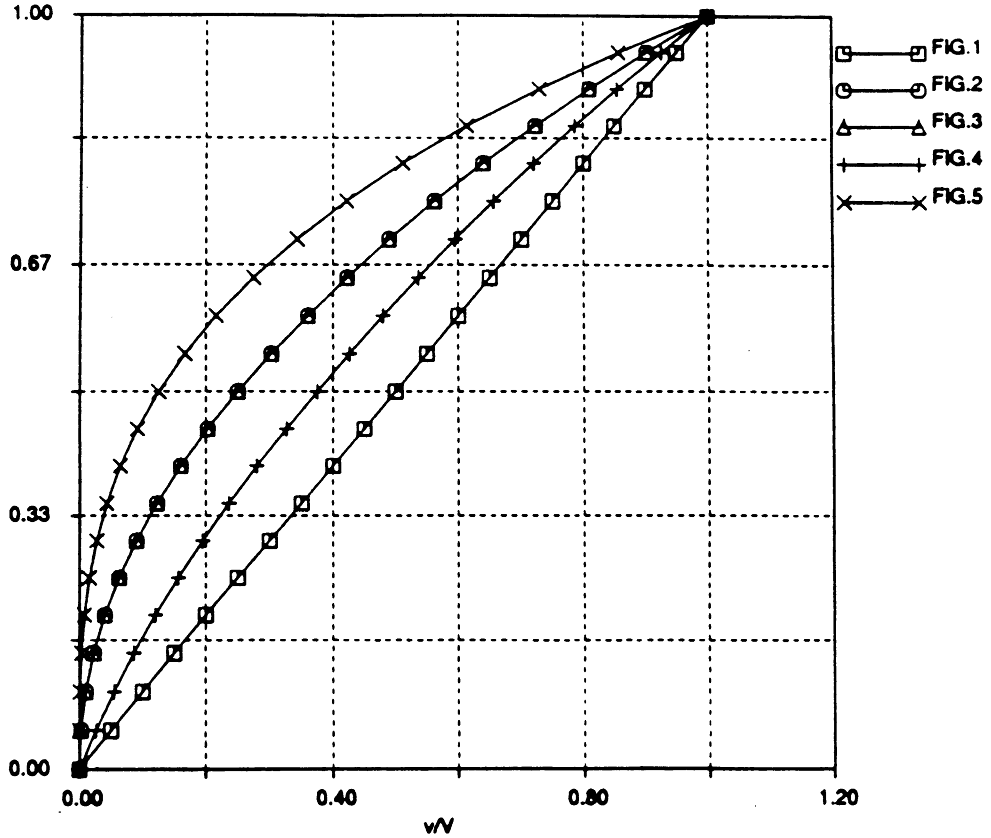

Figure 5-13: Effect of side slope on NENV curves

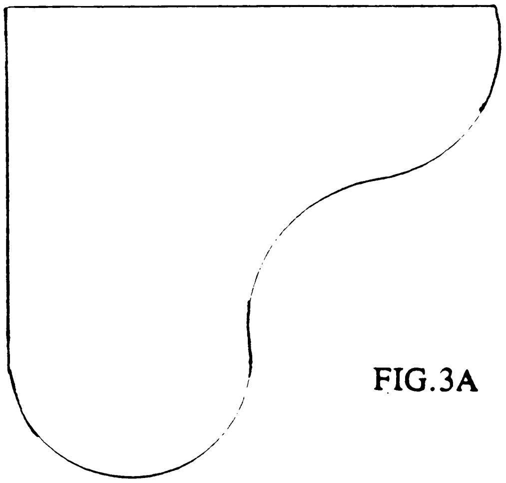

FIG.3A

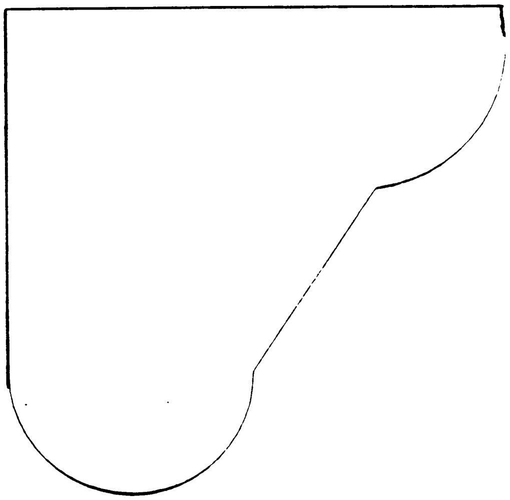

FIG.3B

Figure 5-14: Hypothetical sectional views of shapes chosen to illustrate effect of spacing of contour lines on NENV curves

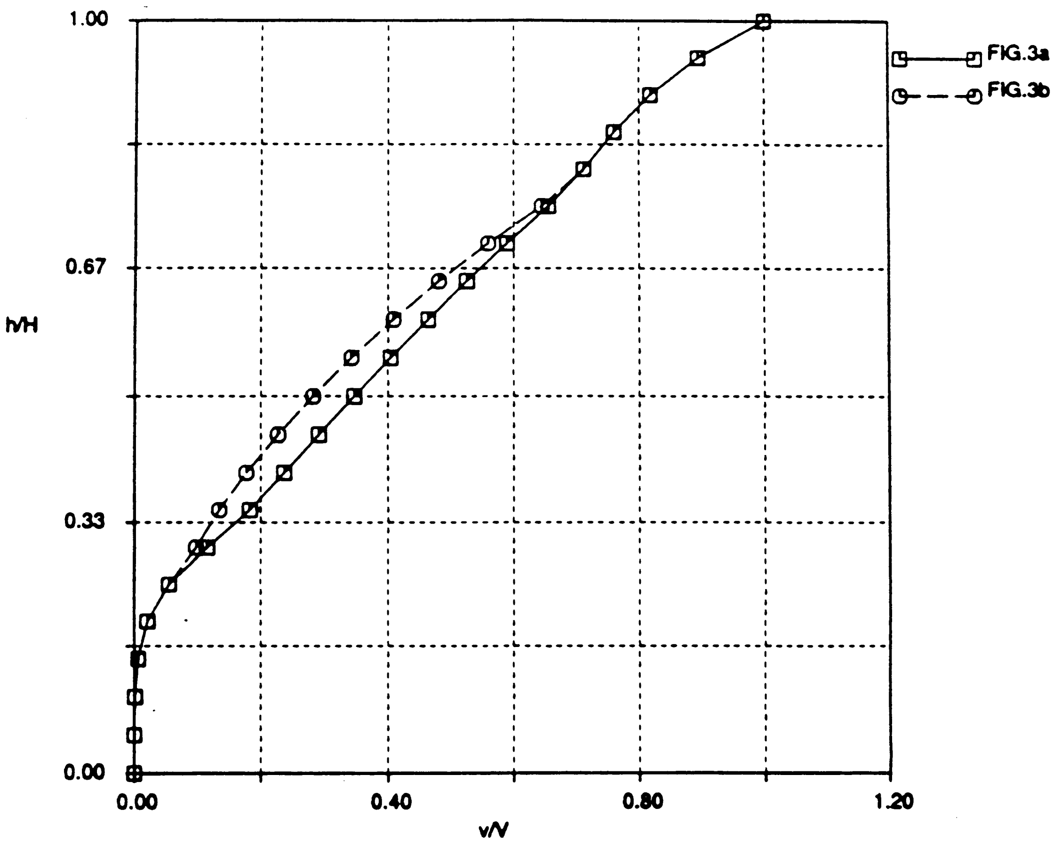

Figure 5-15: Effect of spacing of contour lines on NENV curves

A reservoir is a complex geomorphic landform whose bed is reshaped over the years by sediment deposition. The manner in which the sediment is deposited will depend on a variety of factors which include:

- Topography of the watershed
- Parent material of the soils on the watershed
- Precipitation
- Ability of the soils to retain water
- Vegetation
- Depth of the reservoir
- Reservoir trap efficiency
- Inflow velocity of sediment laden water

The abovementioned factors influence the amount of sedimentation and the distribution of sediments. The NENV curves for a reservoir thus are influenced by these factors; and the uniqueness of the curves for a particular reservoir can be attributed to the these factors also.

5.6: Application of NENV curves

As stated earlier, it was found out that the normalized elevation vs normalized volume (NENV) curves for a particular lake are time invariant. This characteristic property of a lake can be used to estimate the amount of sedimentation in reservoirs if historic data of earlier bathymetric surveys are available. In Iowa, lakes have been surveyed in the past and these data are available with the Iowa Department of Natural Resources (IDNR).

A computer model, based on trapezoidal rule is proposed to compute lake volumes, using NENV curves. For the application of this model the area of the reservoir and the depth at the 5% area contour have to be known and these can be obtained conducting limited reservoir survey. This model was applied to the lakes under study, to test the its applicability.

5.7: NENV Computer Model and terminology used

Before proposing the NENV model and solving the algorithms, it is essential to understand the terminology that has been used in the model.

- 5% lake area contour (a_0): This is the area within the contour which encompasses the deepest 5% of the lake area. Thus if 'A' represents the lake surface area at any instant then,

$$a_0 - 0.05A$$

- lake depth at 5% lake area contour (H): This is the depth of the lake measured from the lake water level to the 5% lake area contour. Thus 'H' equals the difference in the elevation at the lake water level and the 5% lake area contour. Thus if 'B' represents the elevation above the mean sea level at the 5% lake area contour, then 'B+H' will represent the elevation at the lake water level.

- ΔH: The value of ΔH is given by the expression

$$\Delta H=\frac{H}{10}$$

- h_i or h: This is the height measured upwards from the 5% lake area contour elevation. At each stage elevation the value of 'h' is different, and is as shown in Figure 5-16.

Thus

$$h_i=i\Delta H$$

where 'i' takes values from 0 to 10. Figure 5-16 shows the sectional and plan view of a hypothetical reservoir, and explains the various terms used so far in this section.

- a_1, a_2, ..., a_9,: These are the values of the lake area at h = ΔH, 2ΔH, ..., 9ΔH respectively.
- $v^*_{i,i-1}$: This is the volume enclosed between any two elevations or heights (in general between h_i and h_{i-1}. For example, the volume $v^*_{5,6}$ between height $h_5=5\Delta H$ and $h_6=6\Delta H$ is given by

$$v_{5,6}{}^*=\frac{\Delta H}{2}[a_5+a_6]$$

In general,

$$v^*_{i,i-1}=\frac{\Delta H}{2}(a_i+a_{i-1})$$

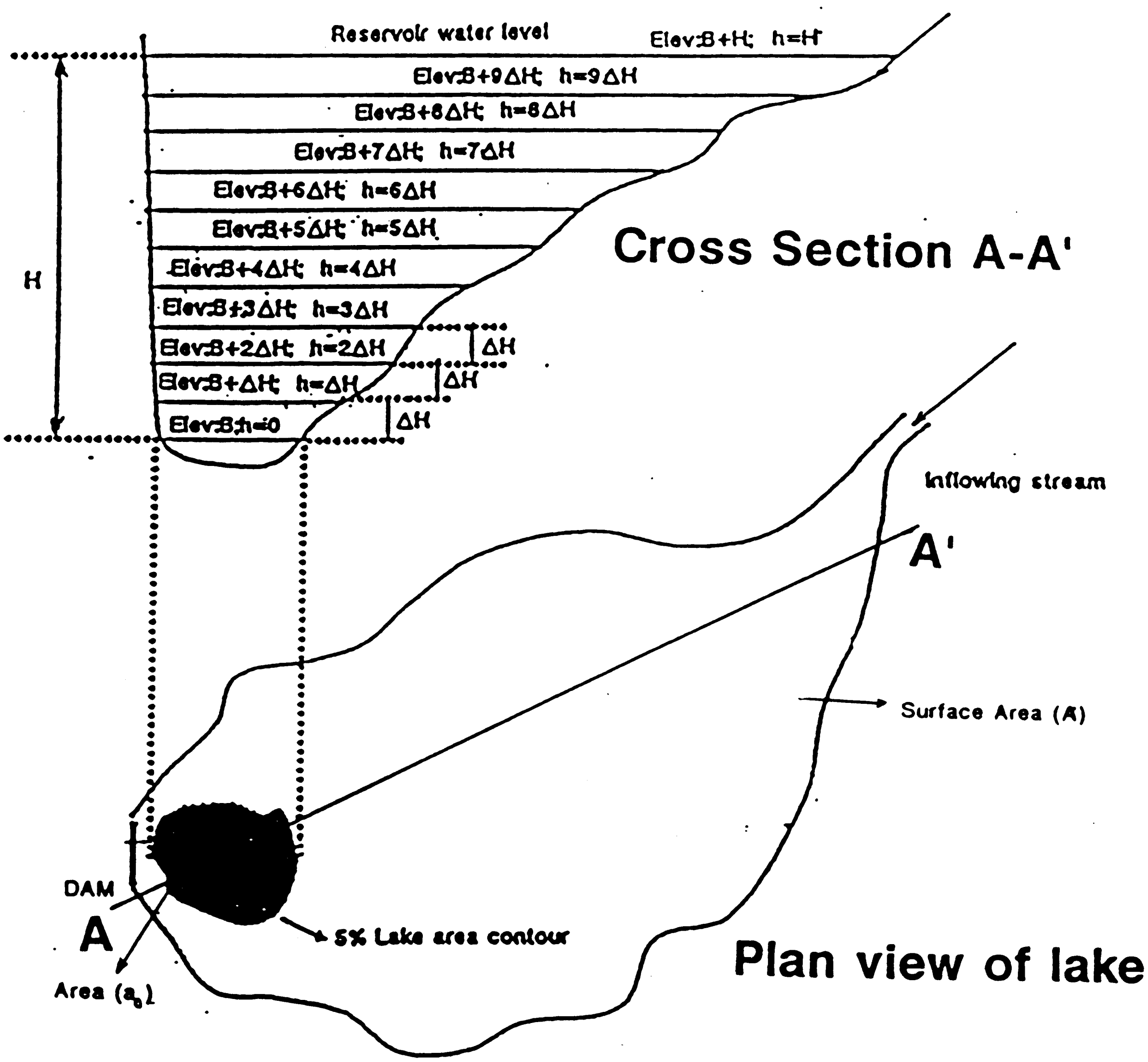

Figure 5-16: Plan and sectional view of a hypothetical reservoir

where i= 1, 2, ..., 10. It should be noted that a_{10} = A = Area of the lake

●v^{**}_i : This is the total volume enclosed below a particular stage. Assuming that the volume of the lake below the 5% lake area contour can be approximated by a triangloid of height ΔH, we have

$$v_0^{**} - \frac{\Delta H}{2} a_0$$

Then, v^{**}_5 up to height h = 5ΔH is given by

$$v_5^{**} - \frac{\Delta H}{2} [a_5 + 2 \sum_0^4 a_i]$$

where i = 0, 1, 2, ..., 9

It should be noted that at i=0, with area=a_0 and volume =v_0^*

$$v_0^* - v_0^{**} - \frac{\Delta H}{2} a_0$$

In general,

$$v_i^{**} = v_{i-1}^{**} + v_{i,i-1}^*$$

where i=1,2, ..., 10

Also,

$$v_j^{**} - \frac{\Delta H}{2} (a_j + 2 \sum_0^{j-1} a_i)$$

where j = 0, 1, 2, ..., 10

- V: This is the total volume of the lake. It is also equal to v^{**}_i at the lake water level, i.e. at 10ΔH. Thus, V = v_{10}^{**}

It should be noted that this expression for the volume of the lake is in terms of unknown areas, a_1, a_2, ..., a_9, which can be obtained using the NENV model.

- x_0, x_1, ..., x_9: These are the ratios of v^{**}_i to the total volume of the lake at stages h/H = 0, 0.1, ..., 0.9 respectively. The values of x_0, x_1, ..., x_9 for the lakes under study were obtained using SURFER, and were found out to be nearly time-invariant. To obtain the values for x_0, x_1, ..., x_9, the historic lake bathymetric maps were digitized and v^{**}_i obtained at elevations B, B+ΔH, ..., B+9ΔH. The v^{**}_i were then normalized with the lake volume (V) to obtain x_0, x_1, ..., x_9. Thus,

$$x_0 = \frac{v_0^{**}@B}{V@B+H}$$

$$x_1 = \frac{v_1^{**}@B+\Delta H}{V@B+H}$$

In general,

$$x_i = \frac{v_i^{**}@B+i\Delta H}{V@B+H}$$

It should be noted that the normalization was done at 5% lake area contour and not the maximum depth. Thus at 5% lake area contour, the lake had some volume and so v^{**}_0 at elevation 'B' and thus x_0 was not equal to zero.

A computer model (NENV model) was developed (as shown in Appendix D) to calculate the lake volume at any time, provided the area of the lake and the lake depth at the 5% lake area contour are known. The program is based on trapezoidal rule, and the basic equations used for the program are as shown in Table D-4 of Appendix D.

The following section gives the algorithm of the program. For a detailed solution of the algorithm refer Appendix D of the thesis.

5.8: NENV model algorithm

The volume of the lake is given by the expression

$$V=\frac{\Delta H}{2}[A+2\sum_{0}^{9}a_i]$$

Thus to compute the volume of the lake it is necessary to solve for a_1, a_2, ..., a_9 In order to achieve this objective, it is necessary to express these variables in terms of some known values. From the aerial photographs we can obtain the value of the lake surface area (A). Also as $a_0 = 0.05$ A, the value of a_0 is also known. From historic bathymetric maps we know x_0, x_1, x_2, ..., x_9. Thus to solve for the variables it is necessary to express the variables in terms of known values 'A' and 'a_0' and ΔH. One approach to this problem is to represent the variables in terms of 'A' and 'a_9'. Then, a_9 can be represented in terms of the known terms: 'A' and 'a_0'.

From Table D-4,

$$x_9 = \frac{x_9}{1} = \frac{a_9+2\sum_{0}^{8} a_i}{A+2\sum_{0}^{9} a_i} \qquad (1)$$

It is possible using "dividendo", to represent the arithmetic series $a_0+a_1+\ldots+a_8$ in terms of A and a_9 (refer to Appendix D for details)

$$\frac{x_9}{1-x_9} = \frac{a_9+2\sum_{0}^{8} a_i}{A+a_9} \qquad (2)$$

Solving for the arithmetic series $a_0+a_1+\ldots+a_8$ we get,

$$\sum_{0}^{8} a_i = 0.5\times\left[\frac{x_9}{1-x_9}A+\frac{2x_9-1}{1-x_9}a_9\right] \qquad (3)$$

Now,

$$x_8 = \frac{a_8+2\sum_{0}^{7} a_i}{A+2\sum_{0}^{9} a_i} = \frac{2\sum_{0}^{7} a_i+a_8+(a_8-a_8)}{A+2\sum_{0}^{8} a_i+2a_9} = \frac{2\sum_{0}^{8} a_i-a_8}{A+2\sum_{0}^{8} a_i+2a_9} \qquad (4)$$

By substituting (3) in (4), we get

$$x_8 = \frac{[\frac{x_9}{1-x_9}A+\frac{2x_9-1}{1-x_9}a_9]-a_8}{A+[\frac{x_9}{1-x_9}A+\frac{2x_9-1}{1-x_9}a_9]+2a_9} \tag{5}$$

Solving for a_8 and combining the terms we get,

$$a_8 = [\frac{x_9-x_8}{1-x_9}A+\frac{-x_8+2x_9-1}{1-x_9}a_9] \tag{6}$$

Similarly by solving we get

$$a_7 = [\frac{-x_9+2x_8-x_7}{1-x-9}A+\frac{-x_7+2x_8-2x_9+1}{1-x_9}a_9] \tag{7}$$

and,

$$a_6 = [\frac{x_9-2x_8+2x_7-x_6}{1-x_9}A+\frac{-x_6+2x_7-2x_8+2x_9-1}{1-x_9}a_9] \tag{8}$$

Also,

$$a_0 = [\frac{x_9-2x_8+2x_7-2x_6+2x_5-2x_4+2x_3-2x_2+2x_1-x_0}{1-x_9}A+$$

$$\frac{-x_0+2x_1-2x_2+2x_3-2x_4+2x_5-2x_6+2x_7-2x_8+2x_9-1}{1-x_9}a_9] \tag{9}$$

Refer to Appendix D for a detailed solution of this algorithm. The equations given above are numbered the same in the appendix for simplicity.

Thus we can express all areas in terms of the known scalers x_0, x_1, ..., x_9 and A and a_9. But since normalization of the elevation is done at 5% lake area contour,

$$a_0 - 0.05A$$

Thus, by substituting in (9) the values of the known scalers x_0, x_1, ..., x_9, A and a_0, we can obtain the value for a_9. Substituting the value of a_9 in all the other equations, we can obtain the other height areas a_8, a_7, ..., a_1. The lake volume can then be calculated using the relationship given by trapezoidal rule.

$$V - \frac{\Delta H}{2}[A + 2\sum_{0}^{9} a_i]$$

5.9: Application of the model

The validity of a model can only be ascertained by applying the model to a real world problem. The NENV model was applied to the lakes under study and gave satisfactory results. As mentioned earlier, the application of this model to a reservoir is to compute the reservoir volume and thus the amount of sedimentation. Limited field data are required to apply the model. These include:

- Area of the reservoir
- Reservoir elevation at 5% area contour

The area of a reservoir can be obtained by taking vertical aerial photographs of the reservoir watershed. The reservoir elevation at the 5% area contour interval can be obtained by taking lake bottom soundings at the "approximate" 5% area contour.

Irregularities in the depths can be disregarded, and the other soundings averaged, to obtain the reservoir depth at the 5% lake area contour. It is possible to know the approximate location of the 5% area contour from the basic knowledge of lake morphology. The 5% lake area contour, for a river dammed lake will be towards the dam, for a glacial lake it will be near the center of the lake, and for an oxbow lake will be towards the outside of the meander loop.

SURFER was used to compute the values for x_0, x_1, ..., x_9, for the three lakes under study. The values obtained are as shown in Tables D-1, D-2, and D-3 of Appendix D. These values were input in the NENV computer program, given in Appendix D, to compute the lake volume.

In case of Lower Pine Lake, the data for 1922 were used to compute the lake volume for 1932, 1950, and 1990. The volume obtained by the NENV model was compared with the volume obtained using SURFER and trapezoidal rule. The results are as shown in Table 5-5.

In case of Union Grove Lake the 1936 data were used to compute the lake volume for 1950 and 1981, and the volume obtained using the NENV model was compared with the volume obtained using SURFER and trapezoidal rule. The results are shown in Table 5-6.

Table 5-5: Comparison of volumes: Lower Pine Lake

Year	Volume (acre-ft)			Variation (%)
	NENV	SURFER	Trapezoidal Rule	
1932	499.14	566.35	533.13	6.38
1950	395.76	432.05	436.8	9.4
1990	303.7	354.68	332.85	8.76

Table 5-6: Comparison of volumes: Union Grove Lake

Year	Volume (acre-ft)			Variation (%)
	NENV	SURFER	Trapezoidal Rule	
1950	966	998.94	994.96	2.91
1981	614.66	662.15	693.1	11.32

It should be noted that the highest percentage variation was obtained for the 1981 lake volume. This is because the NENV curve for 1981 is slightly different from the 1936 curve as seen in Figure 5-6. As mentioned earlier the slight variation between the two curves can be attributed to raising of the dam crest and dredging.

In case of Black Hawk Lake the 1916 data were used to compute lake volume for 1935, and the 1973 data were used to compute the lake volume for 1981. The 1973 data were used instead of the 1916 data as the lake was dredged in 1973 and exhibited a NENV curve which was slightly different from the 1916 curve. The volume obtained was then compared to the volume obtained using SURFER and trapezoidal rule. The results are as shown in Table 5-7.

Table 5-7: Comparison of volumes: Black Hawk Lake

Year	NENV	SURFER	Trapezoidal Rule	Variation (%)
1935	3022.16	3349.08	3300.23	8.43
1981	3594.14	3383.22	3685.17	2.47

Thus, to estimate the lake volume using the NENV model, the data obtained from previous bathymetric maps are used. If a lake has been dredged in the past or the dam crest elevation has been raised then the data obtained from bathymetric maps after the event give better results as seen in the case of Union Grove Lake and Black Hawk Lake. In case of Union Grove Lake, use of data prior to raising of the dam crest gave a larger variation (11.32%), whereas, in case of Black Hawk Lake, use of data obtained after dredging gave better results or a smaller variation (2.47%).

It should be noted that in actual practice, the model may yield results with slightly greater variations. These variations may be due to errors induced in the measurements of:

- Area of the lake

  When the NENV model was applied to the lakes under study, the area was obtained directly from the previous bathymetric maps. In actual practice, the area will have to be obtained by other methods such as aerial photographs, in order to avoid an extensive survey.

- Depth at 5% lake area contour

  When the NENV model was applied to the lakes under study the depth at the 5% lake area was obtained based on previous bathymetric maps. In actual practice, however, the depth obtained may only be at the "approximate" 5% lake area contour.

However, if the errors that can be induced by these two factors are recognized and if care is taken to minimize these errors by accurate measurements, it is possible to apply the NENV model to real world problems.

Although the NENV model could be applied to lakes to calculate the lake volume at any instant, it is essential to have a limited field data. If, however, in future it is possible to estimate the rate of decrease of lake area and lake depth, then it may be possible to apply the NENV model to reservoirs without having to actually conduct any field survey.

5.10: Studied lakes as compared to studied geometrical forms

As is seen from Figure 5-10, the NENV curve for Black Hawk Lake is considerably flat as compared to the curves for the other two lakes. As lakes have complex bed profiles, it is not always possible to compare a lake with a geometric model. Black Hawk Lake could be best compared, among the geometrical shapes studied, to the frustum of a cone, whose frustum height to diameter ratio is very small. The NENV curve for such a geometrical model would be somewhat similar to the combined effect of the NENV curves for a cone and the NENV curve for Fig. 4 in Figure 5-12.

In order to compare Union Grove Lake and Lower Pine Lake to the studied geometrical forms the two lakes have been grouped under one category as both are river dammed lakes. These two lakes can be best compared among the various geometrical shapes studied here, with Fig. 3b of Figure 5-14, wherein the ratio of the lake depth to the length is very small. Also, instead of vertical sides (as assumed for Fig. 3b), the sides of the geometric model will be sloping, similar to Fig. 2 of Figure 5-12. The NENV curve for such a geometrical model would be somewhat similar to the combined effect of the NENV curve for Fig. 3b of Figure 5-14 and the NENV curve for Fig. 2 of Figure 5-12.

CONCLUSION

- The purpose of this study is not a critical evaluation of the proposed literature on sedimentation, but to present what has been proposed by various researchers. However, the time independence concept and stream power theory concept have been evaluated using the NENV model.

- Comparisons of the lake volumes obtained by using SURFER and modified prismoidal rule showed a variation of less than 10%.

- The three lakes under study exhibit characteristic normalized elevation vs normalized volume curves which are time independent, thereby suggesting that the reservoir bed profile adjust over a period of time to the discharge of water and sediment load provided by the drainage basin.

- The NENV model can be used to predict the volume of sedimentation and thus the sedimentation at any instant if the area of the lake and the reservoir depth at the 5% area contour is known.

- Comparison of lake volumes obtained by using NENV model and the trapezoidal rule showed a variation of less than 12% for the three lakes under study.

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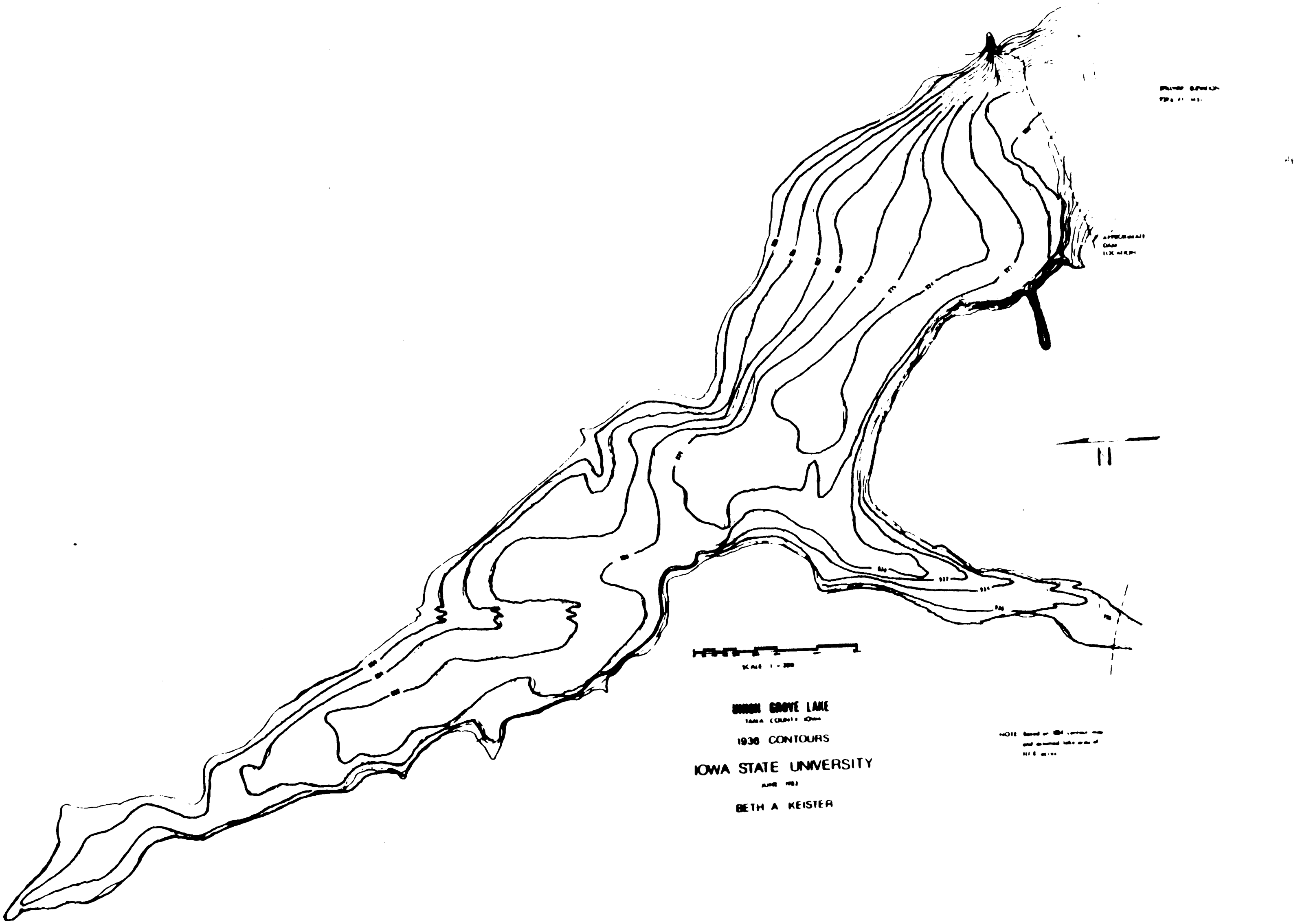
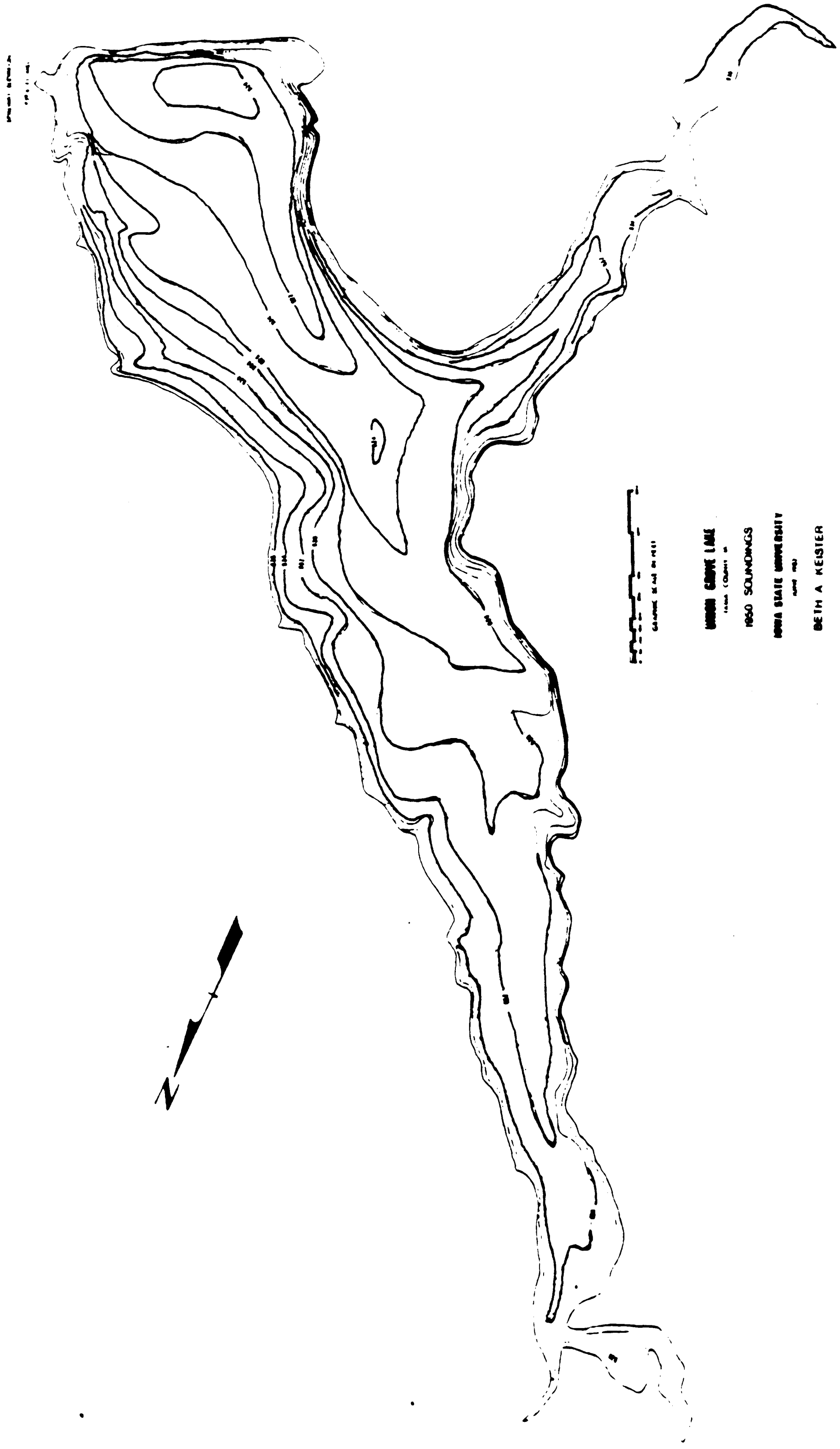
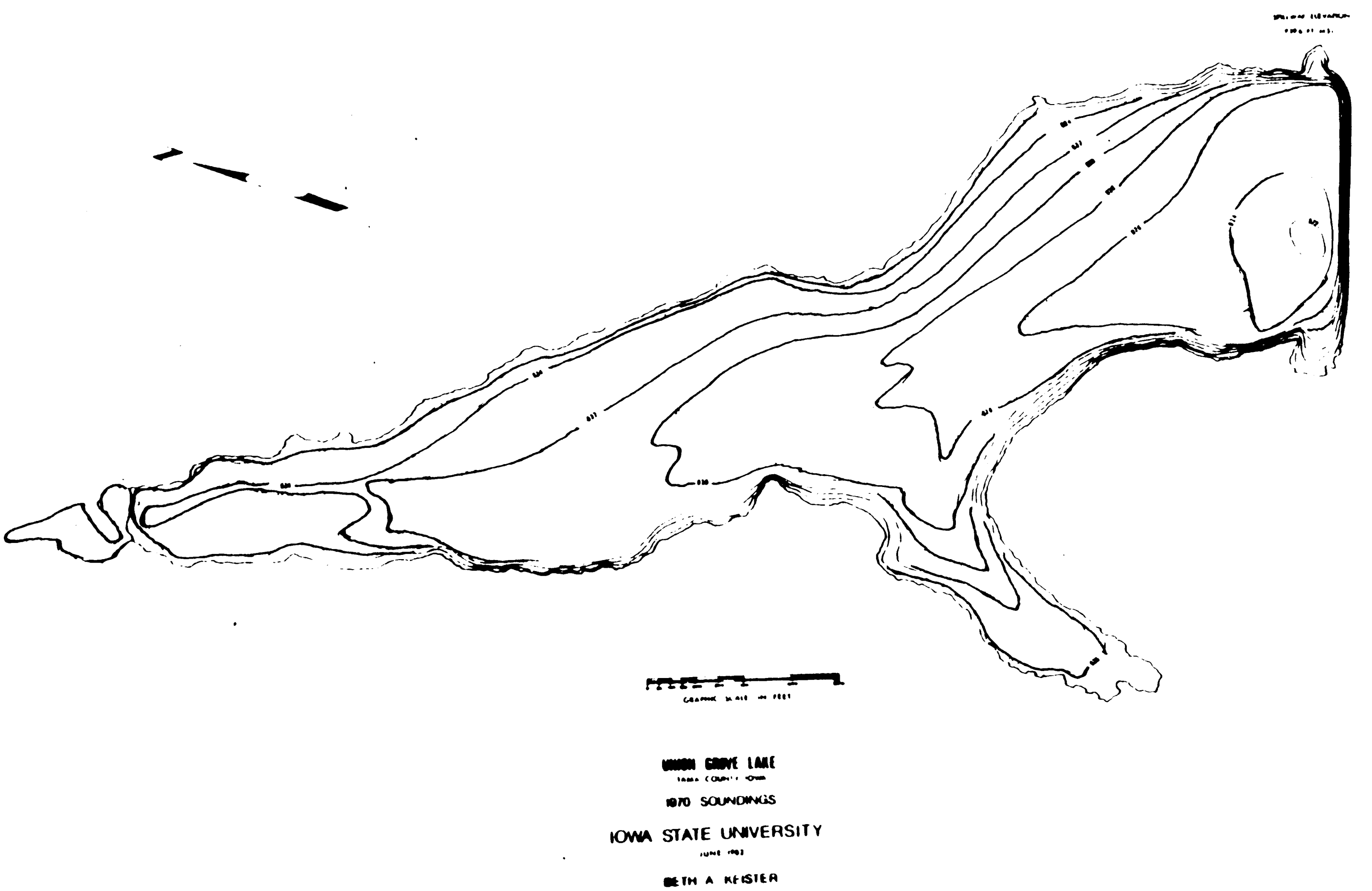
APPENDIX A: LAKE BATHYMETRIC MAPS



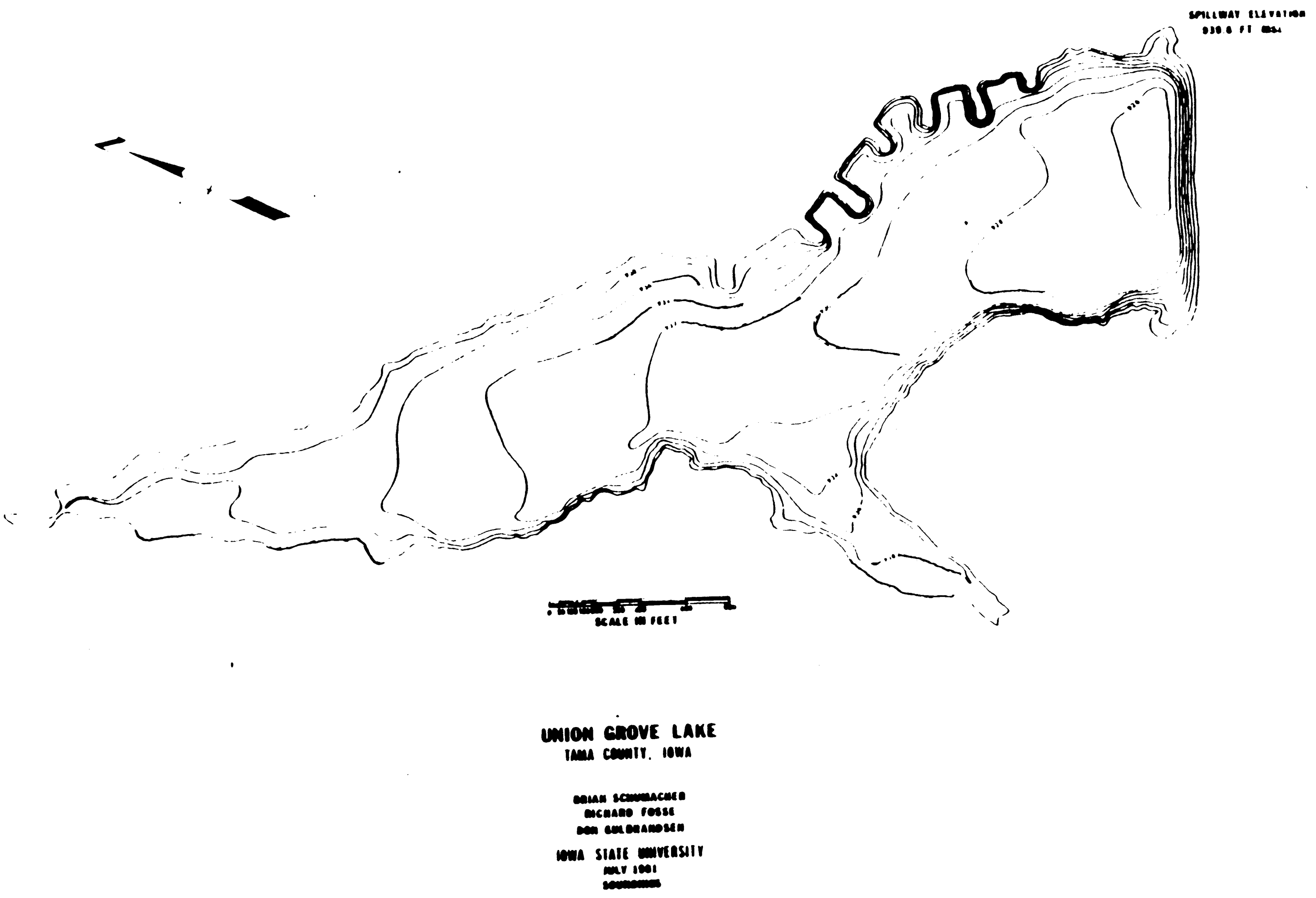
Map A-1: Union Grove Lake - Lake Bathymetric map (1936)



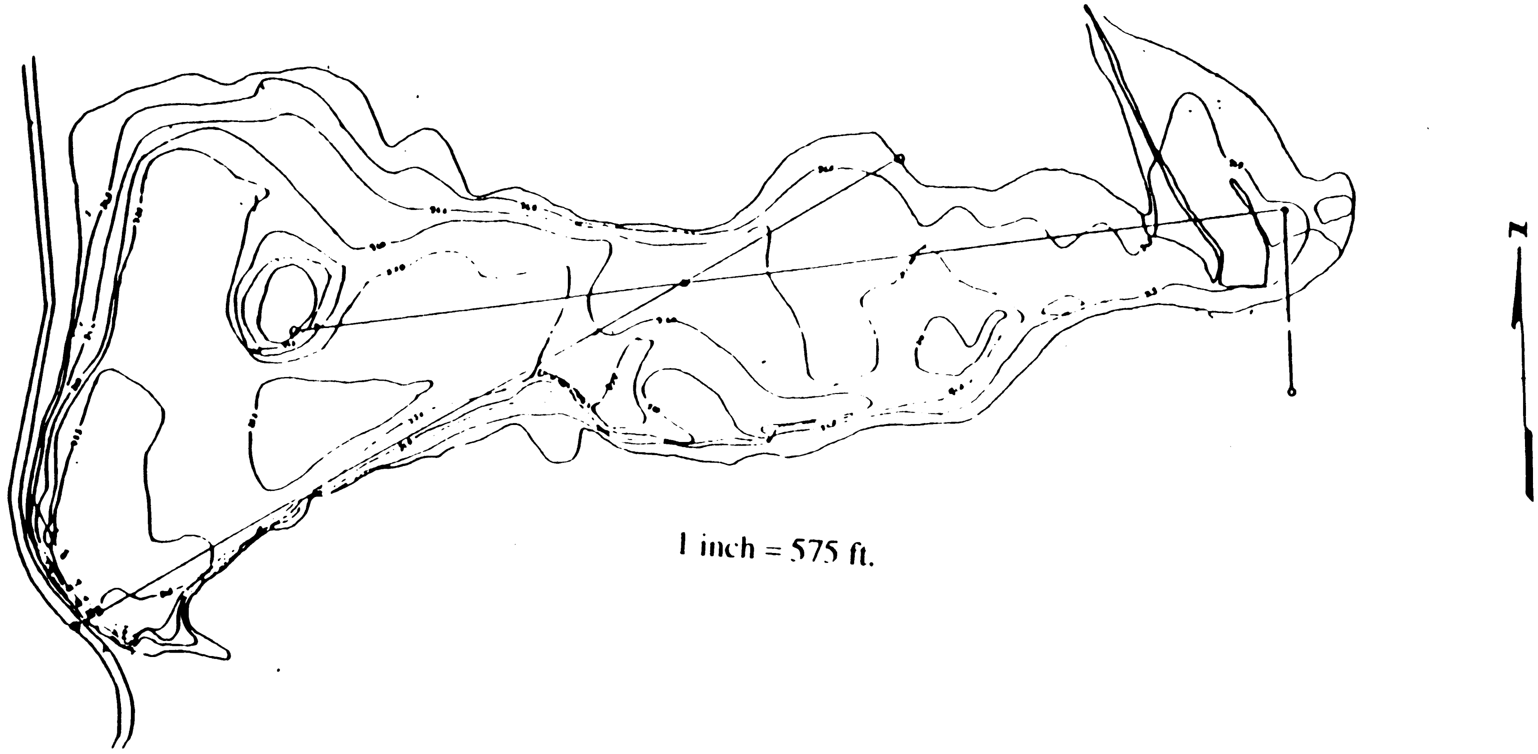
Map A-2: Union Grove Lake - Lake bathymetric Map, (1950)



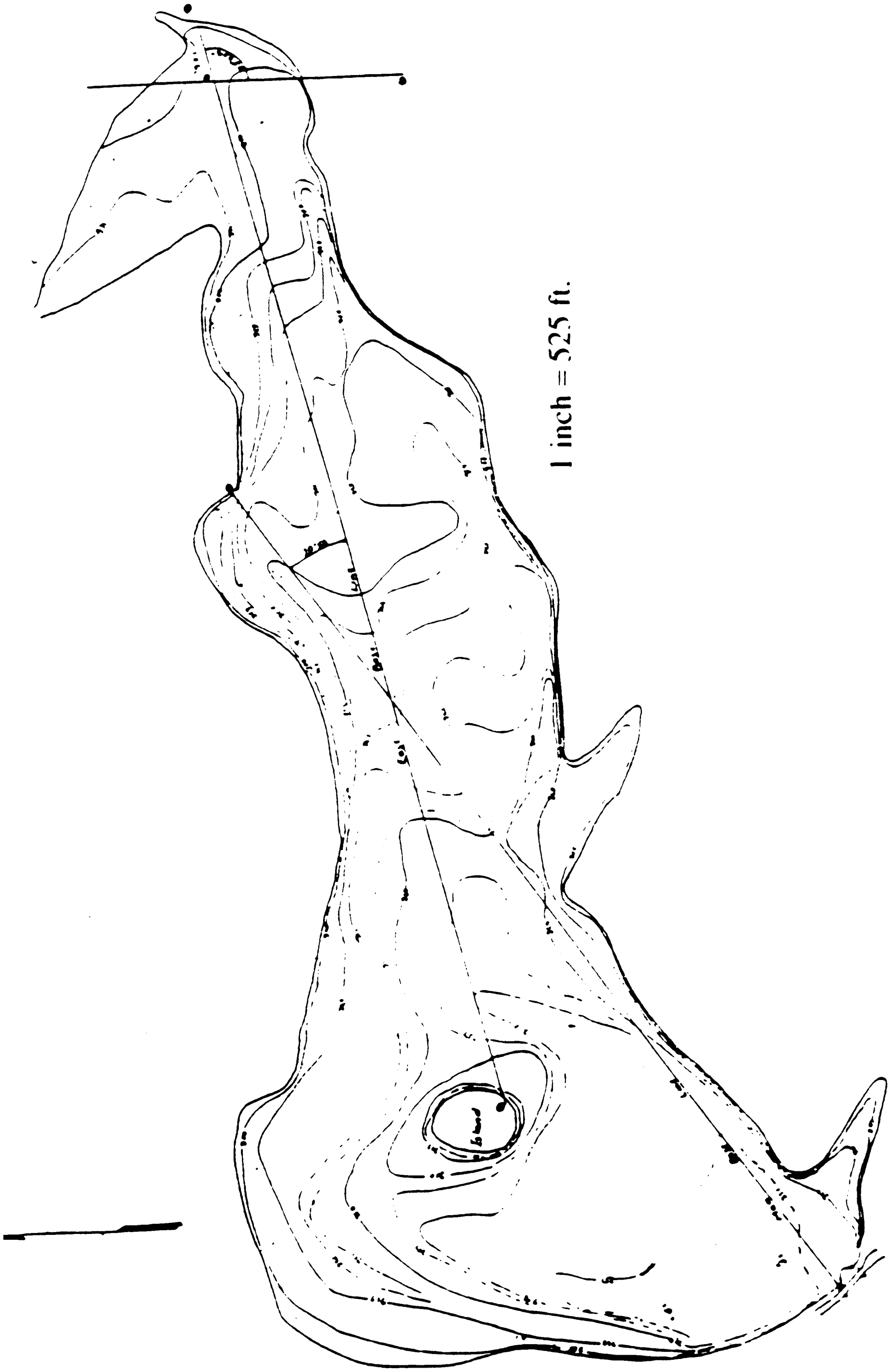
Map A-3: Union Grove Lake - Lake bathymetric Map, (1970)



Map A-4: Union Grove Lake - Lake bathymetric Map, (1981)

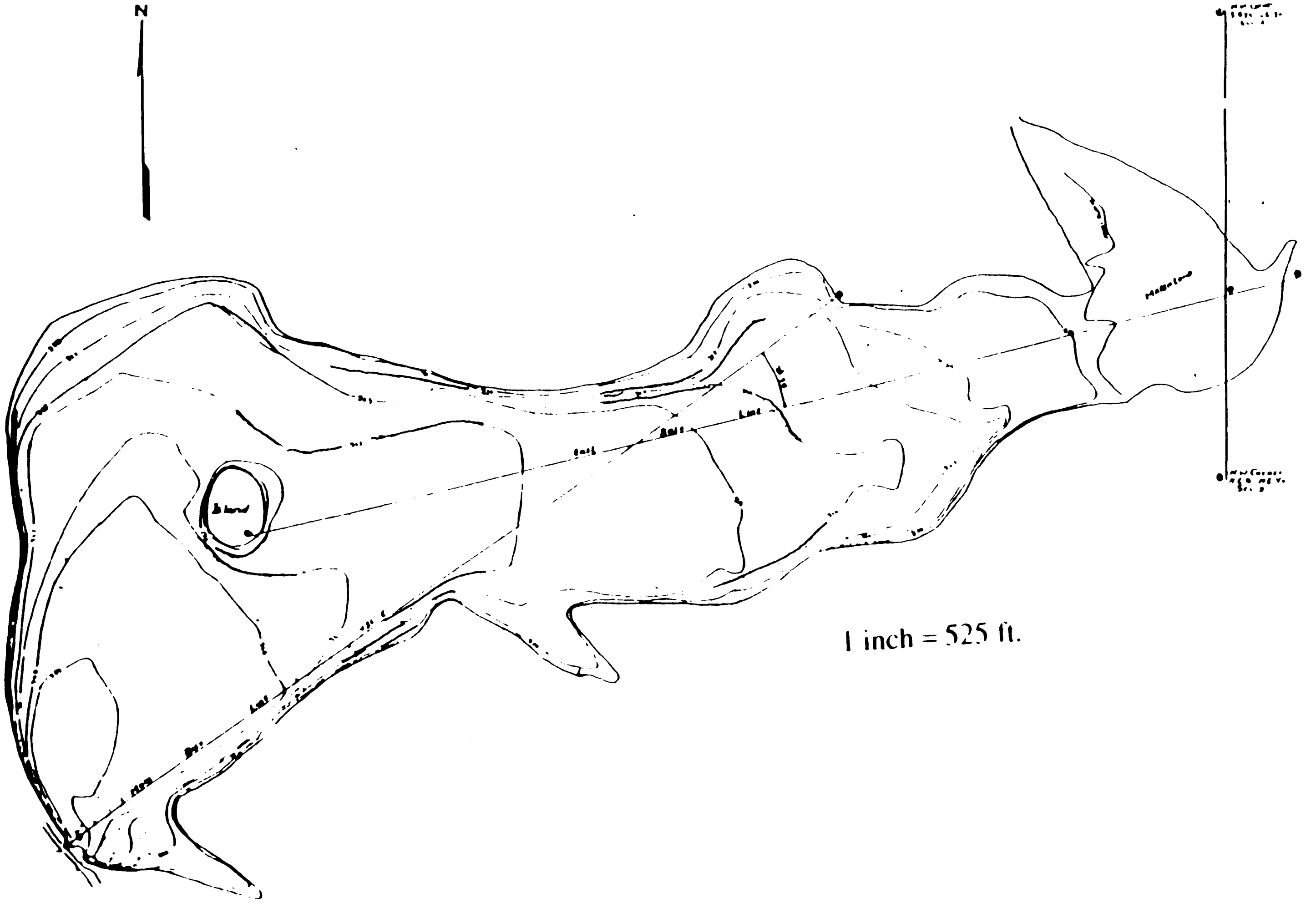


Map A-5: Lower Pine Lake - Lake bathymetric Map, (1922)

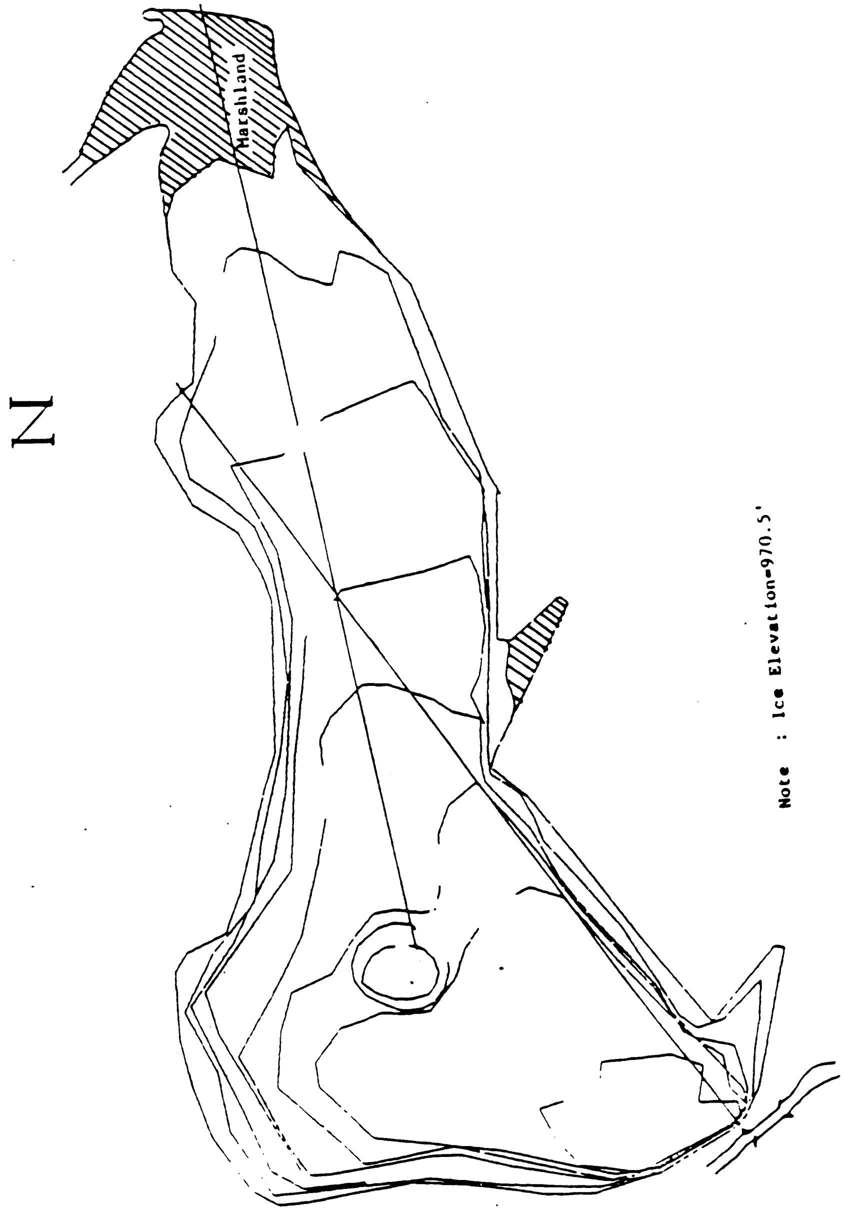


1 inch = 525 ft.

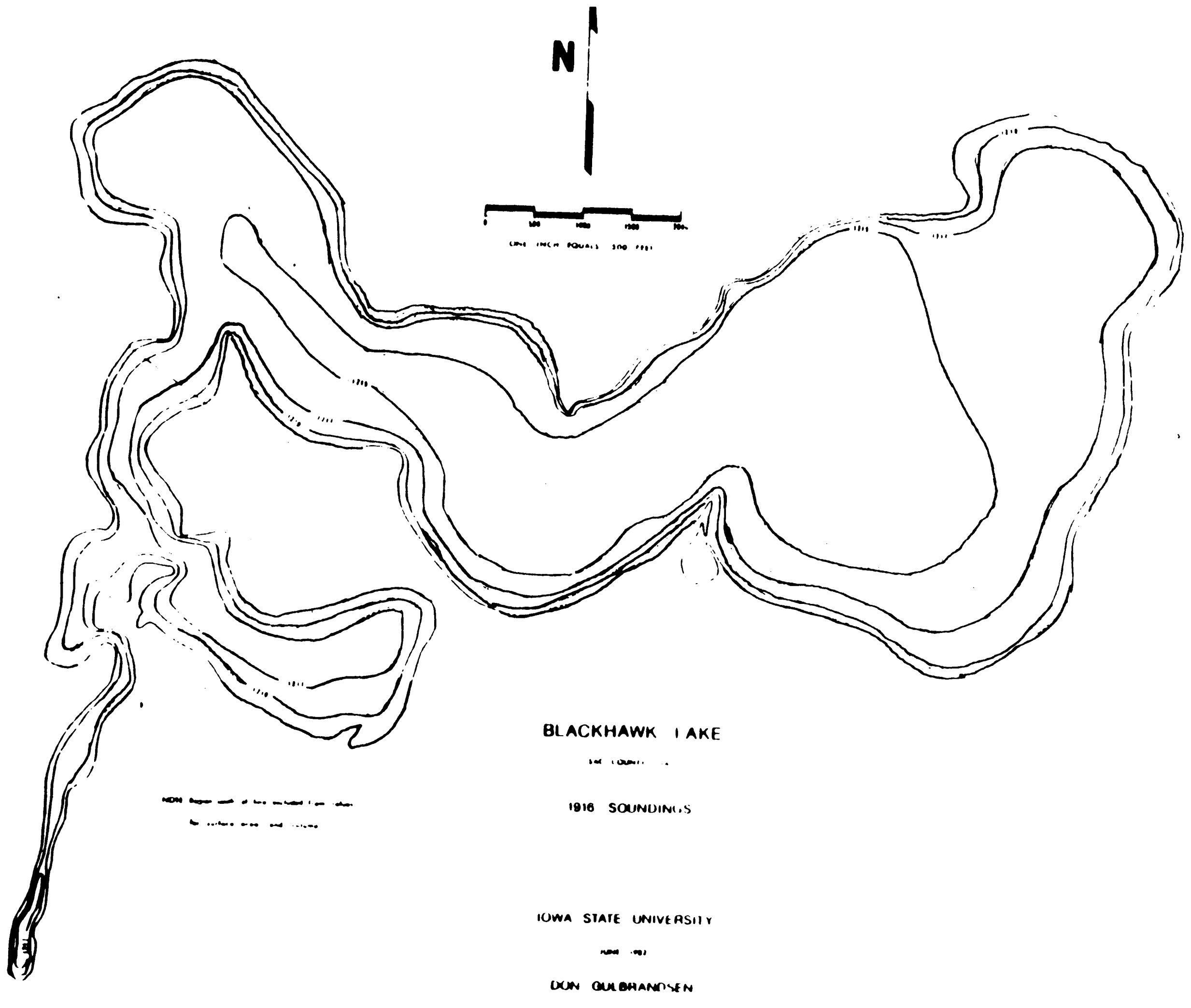
Map A-6: Lower Pine Lake - Lake bathymetric Map, (1932)



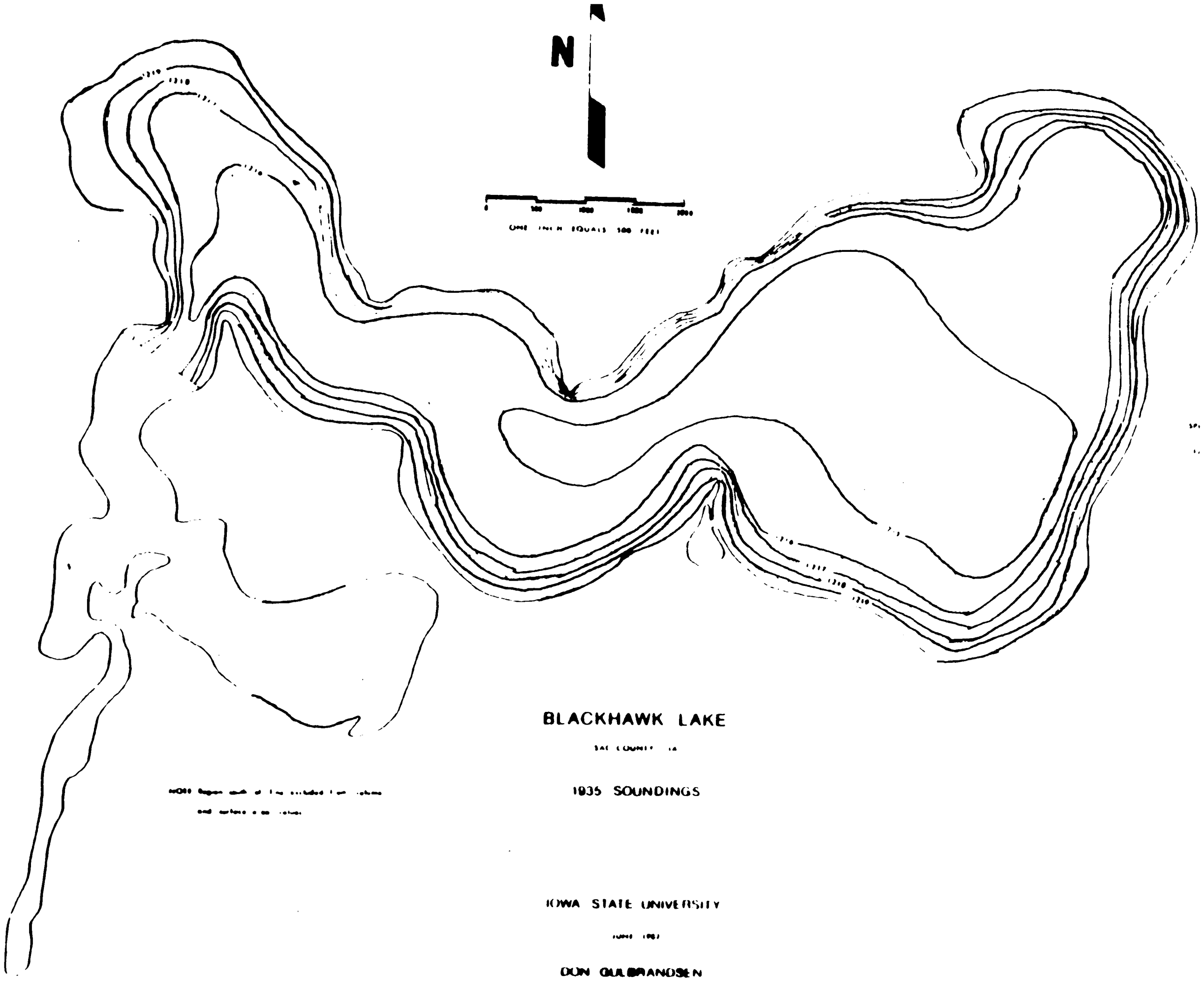
Map A-7: Lower Pine Lake - Lake bathymetric Map, (1950)



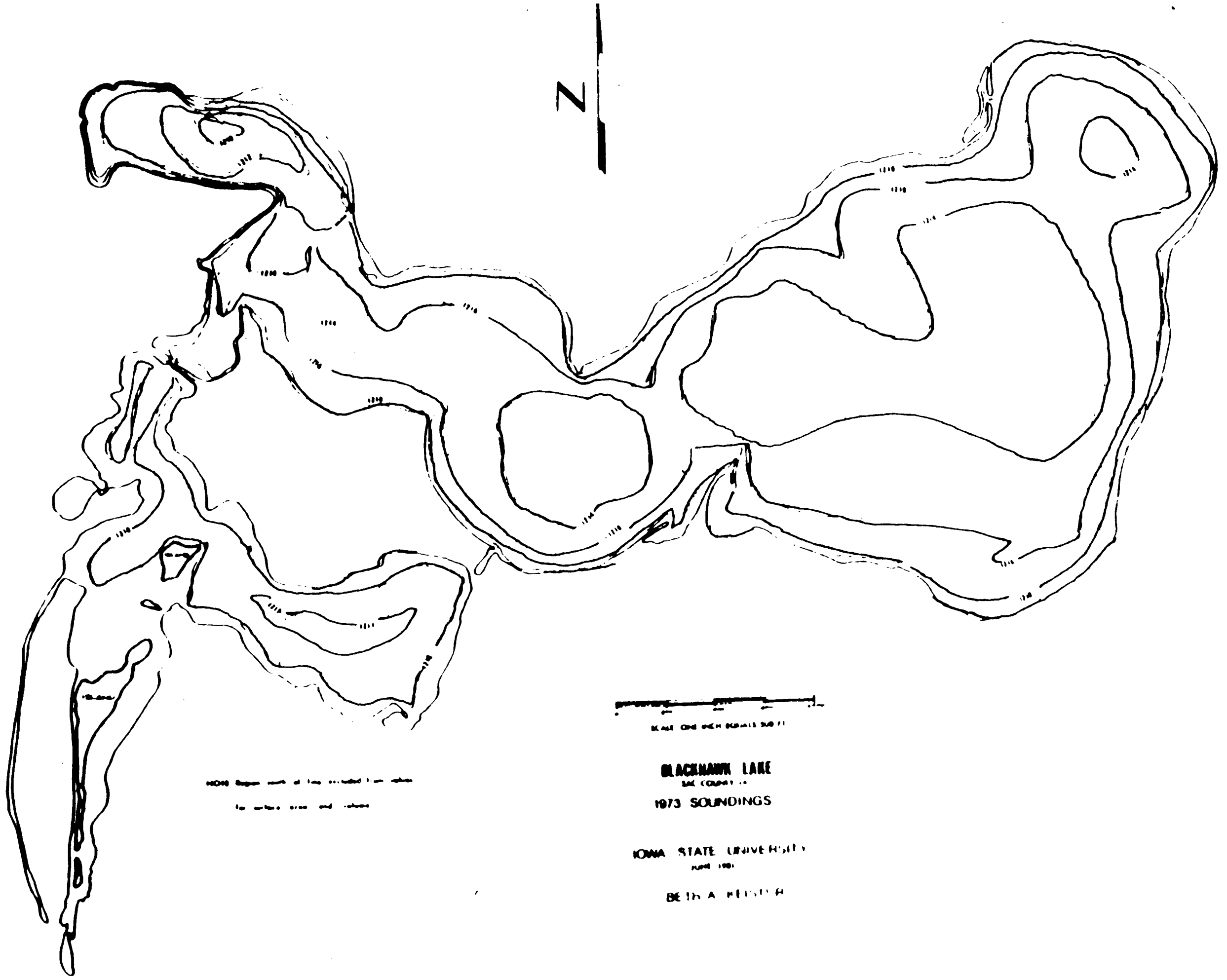
Map A-8: Lower Pine Lake - Lake bathymetric Map, (1990)



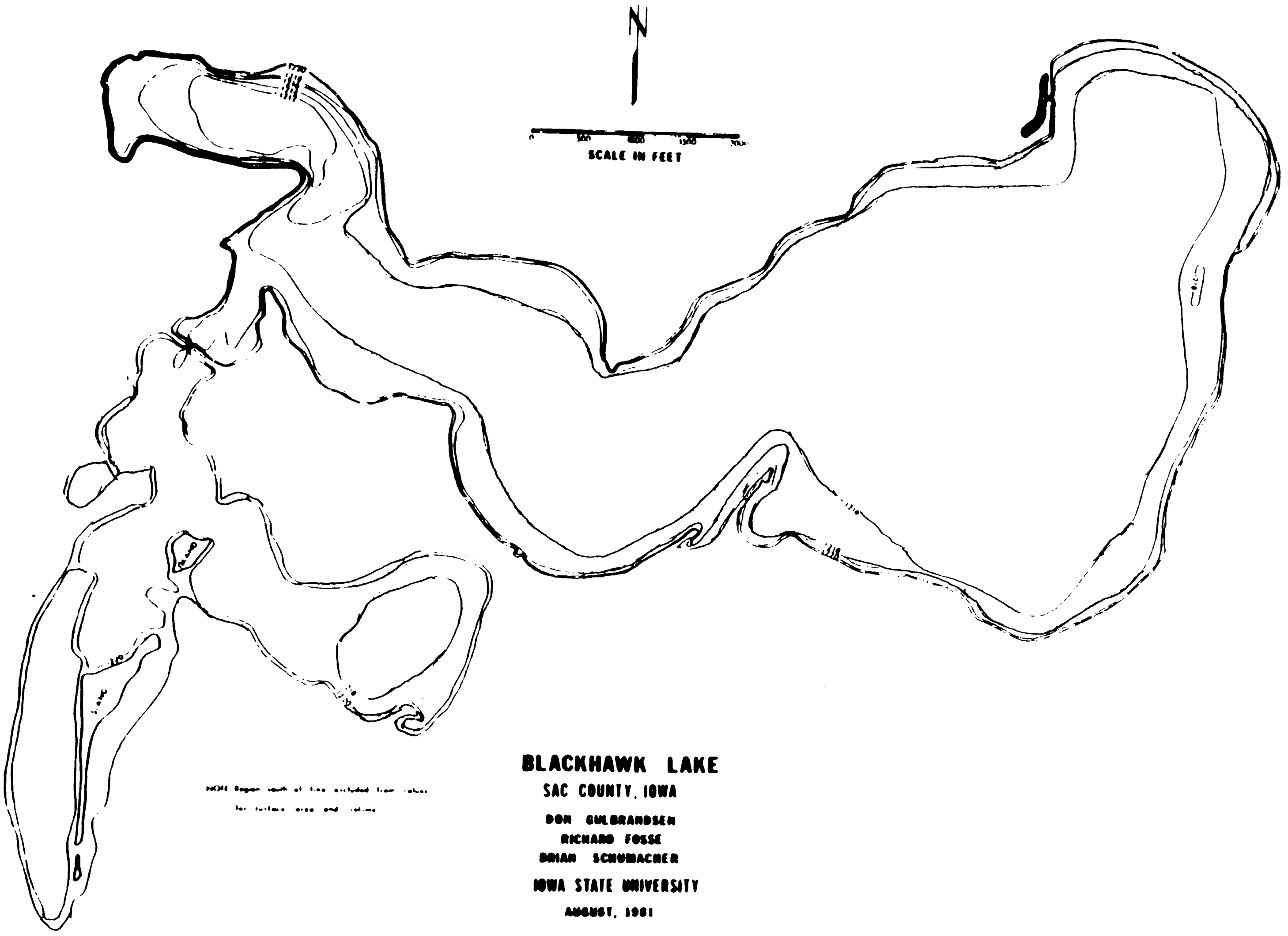
Map A-9: Black Hawk Lake - Lake bathymetric Map, (1916)



Map A-10: Black Hawk Lake - Lake bathymetric Map, (1935)



Map A-11: Black Hawk Lake - Lake bathymetric Map, (1973)



Map A-12: Black Hawk Lake - Lake bathymetric Map, (1981)

APPENDIX B: VOLUME-ELEVATION RELATIONSHIP

Table B-1: Union Grove Lake - Elevation & volume relationship, (1936)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Volume (acre-ft)	Normalized Volume(%)
937.6	0.0	100.00	796.62	100.00
936.0	-1.6	90.36	598.93	75.18
934.0	-3.6	78.31	405.84	50.95
932.0	-5.6	66.27	256.09	32.15
930.0	-7.6	54.22	152.67	19.16
928.0	-9.6	42.17	82.34	10.34
926.0	-11.6	30.12	37.78	4.74
924.0	-13.6	18.07	14.19	1.78
922.0	-15.6	6.02	2.67	0.34
921.0	-16.6	0.00	0.61	0.08

Table B-2: Union Grove Lake - Elevation & volume relationship, (1950)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Volume (acre-ft)	Normalized Volume(%)
939.6	0.0	100.00	998.94	100.00
938.0	-1.6	90.91	774.53	77.54
937.6	-2.0	88.64	724.98	72.57
936.0	-3.6	79.55	548.19	54.88
934.0	-5.6	68.18	369.05	36.94
932.0	-7.6	56.82	230.29	23.05
930.0	-9.6	45.45	130.58	13.07
928.0	-11.6	34.09	63.63	6.37
926.0	-13.6	22.73	24.78	2.48
924.0	-15.6	11.36	5.52	0.55
922.0	-17.6	0.00	0.29	0.03

Table B-3: Union Grove Lake - Elevation & volume relationship, (1970)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Volume (acre-ft)	Normalized Volume(%)
939.6	0.0	100.00	836.12	100.00
937.6	-2.0	87.18	598.91	71.63
936.0	-3.6	76.92	459.95	55.01
934.0	-5.6	64.10	310.37	37.12
932.0	-7.6	51.28	205.77	24.61
930.0	-9.6	38.46	118.23	14.14
928.0	-11.6	25.64	60.28	7.21
926.0	-13.6	12.82	24.75	2.96
924.0	-15.6	0.00	0.18	0.02

Table B-4: Union Grove Lake - Elevation & volume relationship, (1981)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Volume (acre-ft)	Normalized Volume(%)
937.6	0.0	100.00	662.15	100.00
936.0	-1.6	88.24	495.90	74.89
934.0	-3.6	73.53	331.26	50.03
932.0	-5.6	58.82	201.65	30.45
930.0	-7.6	44.12	108.19	16.34
928.0	-9.6	29.41	47.43	7.16
926.0	-11.6	14.71	11.75	1.77
924.0	-13.6	0.00	0.17	0.03

Table B-5: Lower Pine Lake - Elevation & volume relationship, (1922)

Elevation (ft)	Depth (ft)	Normalised Elevation	Volume (acre-ft)	Normalised Volume
970.5	0	100.00	680.90	100.00
969	-1.5	90.32	526.17	77.28
967	-3.5	77.42	390.04	57.28
965	-5.5	64.52	272.04	39.95
963	-7.5	51.61	173.97	25.55
961	-9.5	38.71	95.53	14.03
959	-11.5	25.81	44.62	6.55
958	-12.5	19.35	27.86	4.09
957	-13.5	12.90	15.83	2.32
956	-14.5	6.45	7.26	1.07
955	-15.5	0.00	1.82	0.27

Table B-6: Lower Pine Lake - Elevation & volume relationship, (1932)

Elevation (m.s.l.)	Depth (ft)	Normalized Depth(%)	Volume (acre-ft)	Normalized Volume(%)
971	0.00	0.00	586.35	100.00
969	-2.00	-14.29	411.16	70.12
967	-4.00	-28.57	285.81	48.74
965	-6.00	-42.86	184.83	31.52
963	-8.00	-57.14	112.00	19.10
961	-10.00	-71.43	60.26	10.28
959	-12.00	-85.71	29.51	5.03
958	-13.00	-92.86	20.11	3.43
957	-14.00	-100.00	13.83	2.36

Table B-7: Lower Pine Lake - Elevation & volume relationship, (1950)

Elevation (ft)	Depth (ft)	Normalized Elevation (%)	Volume (acre-ft)	Normalized Volume (%)
971	0	100.00	516.52	100.00
969	-2	83.33	353.54	68.45
967	-4	66.67	223.60	43.29
965	-6	50.00	126.01	24.40
963	-8	33.33	58.08	11.24
962	-9	25.00	37.95	7.35
961	-10	16.67	23.42	4.53
960	-11	8.33	12.17	2.36
959	-12	0.00	4.59	0.89

Table B-8: Lower Pine Lake - Elevation & volume relationship, (1990)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation (%)	Volume (acre-ft)	Normalized Volume(%)
970.5	0.0	100.0	354.68	100.00
969.5	-1.0	90.0	286.27	80.71
968.5	-2.0	80.0	226.88	63.97
967.5	-3.0	70.0	175.32	49.43
966.5	-4.0	60.0	130.88	36.90
965.5	-5.0	50.0	92.58	26.10
964.5	-6.0	40.0	60.47	17.05
963.5	-7.0	30.0	35.49	10.01
962.5	-8.0	20.0	18.69	5.27
961.5	-9.0	10.0	7.74	2.18
960.5	-10.0	0.0	1.47	0.41

Table B-9: Black Hawk Lake - Elevation & volume relationship, (1916)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Volume (acre-ft)	Normalized Volume(%)
1220.5	0.0	100.00	3994.01	100.00
1220.0	-0.5	90.91	3333.76	83.47
1219.0	-1.5	72.73	2577.65	64.54
1218.0	-2.5	54.55	1890.87	47.34
1217.0	-3.5	36.36	1272.81	31.87
1216.0	-4.5	18.18	730.42	18.29
1215.0	-5.5	0.00	310.46	7.77

Table B-10: Black Hawk Lake - Elevation & volume relationship, (1935)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Volume (acre-ft)	Normalized Volume(%)
1220.5	0.0	100.00	3349.08	100.00
1219.0	-1.5	75.00	1898.05	56.67
1218.0	-2.5	58.33	1297.63	38.75
1217.0	-3.5	41.67	784.90	23.44
1216.0	-4.5	25.00	372.93	11.14
1215.0	-5.5	8.33	87.63	2.62
1214.5	-6.0	0.00	18.94	0.57

Table B-11: Black Hawk Lake - Elevation & volume relationship, (1973)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Volume (acre-ft)	Normalized Volume(%)
1220.5	0.0	100.00	3813.25	100.00
1220.0	-0.5	92.31	3198.94	83.89
1219.0	-1.5	76.92	2490.43	65.31
1218.0	-2.5	61.54	1842.56	48.32
1217.0	-3.5	46.15	1253.03	32.86
1216.0	-4.5	30.77	736.72	19.32
1215.0	-5.5	15.38	339.00	8.89
1214.0	-6.5	0.00	73.98	1.94

Table B-12: Black Hawk Lake - Elevation & volume relationship, (1981)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Volume (acre-ft)	Normalized Volume(%)
1220.5	0.0	100.00	3383.22	100.00
1220.0	-0.5	90.91	2874.38	84.96
1219.0	-1.5	72.73	2238.00	66.15
1218.0	-2.5	54.55	1675.03	49.51
1217.0	-3.5	36.36	1153.68	34.10
1216.0	-4.5	18.18	665.48	19.67
1215.0	-5.5	0.00	243.25	7.19

APPENDIX C: AREA-ELEVATION RELATIONSHIP

Table C-1: Union Grove Lake - Elevation & area relationship, (1936)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Area (acre)	Normalized Area(%)
937.60	0.00	100.00	118.04	100.00
936.00	-1.60	90.91	105.40	89.29
934.00	-3.60	79.55	87.67	74.27
932.00	-5.60	68.18	67.44	57.13
930.00	-7.60	56.82	45.76	38.77
928.00	-9.60	45.45	32.28	27.35
926.00	-11.60	34.09	20.24	17.15
924.00	-13.60	22.73	10.82	9.17
922.00	-15.60	11.36	5.90	5.00
920.00	-17.60	0.00	1.51	1.28

Table C-2: Union Grove Lake - Elevation & area relationship, (1950)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Area (acre)	Normalized Area(%)
939.6	0.0	100.00	129.55	100.00
937.6	-2.0	88.64	105.30	81.28
934.0	-5.6	68.18	84.81	65.47
932.0	-7.6	56.82	62.41	48.17
930.0	-9.6	45.45	46.71	36.06
928.0	-11.6	34.09	31.16	24.05
926.0	-13.6	22.73	17.63	13.61
924.0	-15.6	11.36	8.19	6.32
922.0	-17.6	0.00	1.75	1.35

Table C-3: Union Grove Lake - Elevation & area relationship, (1970)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Area (acre)	Normalized Area(%)
939.6	0.0	100.00	116.85	100.00
937.6	-2.0	88.64	101.21	86.62
934.0	-5.6	68.18	81.46	69.71
932.0	-7.6	56.82	61.01	52.21
930.0	-9.6	45.45	45.54	38.97
928.0	-11.6	34.09	30.68	26.26
926.0	-13.6	22.73	17.05	14.59
924.0	-15.6	11.36	7.85	6.72
922.0	-17.6	0.00	0.98	0.84

Table C-4: Union Grove Lake - Elevation & area relationship, (1981)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Area (acre)	Normalized Area(%)
939.6	0.0	100.00	106.62	100.00
938.0	-1.6	88.24	94.39	88.53
936.0	-3.6	73.53	79.76	74.81
934.0	-5.6	58.82	59.99	56.27
932.0	-7.6	44.12	38.14	35.77
930.0	-9.6	29.41	24.83	23.29
928.0	-11.6	14.71	12.12	11.37
926.0	-13.6	0.00	2.38	2.23

Table C-5: Lower Pine Lake - Elevation & area relationship, (1922)

Elevation (ft)	Depth (ft)	Normalized Elevation(%)	Area (acre)	Normalized Area(%)
971	0	100.00	69.63	100.00
965	-6	62.50	56.22	80.74
962	-9	43.75	41.01	58.89
960	-11	31.25	29.56	42.46
958	-13	18.75	20.47	29.40
955	-16	0.00	5.95	8.55

Table C-6: Lower Pine Lake - Elevation & area relationship, (1932)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation (%)	Area (acre)	Normalized Area (%)
971.4	0.00	100.00	68.44	100.00
970	-1.40	90.28	60.92	89.01
968	-3.40	76.39	54.36	79.43
965	-6.40	55.56	44.24	64.65
962	-9.40	34.72	30.09	43.96
960	-11.40	20.83	21.61	31.57
958	-13.40	6.94	12.34	18.03
957	-14.40	0.00	3.90	5.70

Table C-7: Lower Pine Lake - Elevation & area relationship, (1950)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Area (acre)	Normalized Area(%)
970.5	0	100.00	63.27	100.00
970	-0.5	95.65	59.25	93.64
967	-3.5	69.57	49.05	77.53
965	-5.5	52.17	38.52	60.88
963	-7.5	34.78	27.19	42.97
961	-9.5	17.39	14.43	22.80
960	-10.5	8.70	10.00	15.81
959	-11.5	0.00	1.94	3.06

Table C-8: Lower Pine Lake - Elevation & area relationship, (1990)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation (%	Area (acre)	Normalized Area(%)
970.5	0.0	100.0	63.50	100.00
969	-1.5	84.2	51.78	81.54
964	-6.5	31.6	28.43	44.77
963	-7.5	21.1	19.95	31.42
962	-8.5	10.5	13.73	21.62
961	-9.5	0.0	2.85	4.49

Table C-9: Black Hawk Lake - Elevation & area relationship, (1916)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Area (acre)	Normalized Area(%)
1220.5	0.0	100.00	799.26	100.00
1219.0	-1.5	76.92	698.31	87.37
1217.0	-3.5	46.15	566.36	70.86
1215.0	-5.5	15.38	250.73	31.37
1214.5	-6.0	7.69	89.28	11.17
1214.0	-6.5	0.00	14.23	1.78

Table C-10: Black Hawk Lake - Elevation & area relationship, (1935)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Area (acre)	Normalized Area(%)
1220.5	0.0	100.00	791.12	100.00
1219.0	-1.5	77.94	728.44	92.08
1218.0	-2.5	63.24	668.80	84.54
1217.0	-3.5	48.53	605.36	76.52
1216.0	-4.5	33.82	478.64	60.50
1215.5	-5.0	26.47	321.68	40.66
1215.0	-5.5	19.12	178.08	22.51
1214.7	-5.8	14.71	111.24	14.06

Table C-11: Black Hawk Lake - Elevation & area relationship, (1973)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Area (acre)	Normalized Area(%)
1220.5	0.0	100.00	770.83	100.00
1218.0	-2.5	70.59	688.84	89.36
1216.0	-4.5	47.06	520.68	67.55
1214.0	-6.5	23.53	235.92	30.61
1212.0	-8.5	0.00	12.60	1.63

Table C-12: Black Hawk Lake - Elevation & area relationship, (1981)

Elevation (m.s.l.)	Depth (ft)	Normalized Elevation(%)	Area (acre)	Normalized Area(%)
1220.5	0.0	100.00	763.40	100.00
1220.0	-0.5	94.12	751.12	98.39
1218.0	-2.5	70.59	662.99	86.85
1216.0	-4.5	47.06	508.12	66.56
1212.0	-8.5	0.00	14.51	1.90

APPENDIX D

Table D-1: Normalized elevation and normalized volume values for Lower Pine Lake

h/H	1922	1932	1950	1990
	v/V	v/V	v/V	v/V
0.0	0.002667	0.024415	0.009122	0.004100
0.1	0.017711	0.041401	0.027413	0.021800
0.2	0.045325	0.070130	0.055736	0.052700
0.3	0.090346	0.113721	0.094883	0.100100
0.4	0.155522	0.176438	0.158186	0.170500
0.5	0.255498	0.256957	0.244430	0.261000
0.6	0.368508	0.358543	0.349913	0.369000
0.7	0.499875	0.484724	0.479335	0.494300
0.8	0.649626	0.631609	0.630487	0.639700
0.9	0.815659	0.801287	0.803473	0.807100
1.0	1.000000	1.000000	1.000000	1.000000

Table D-2: Normalized elevation and normalized volume values for Union Grove Lake

h/H	1936 v/V	1981 v/V
0.0	0.002581	0.000260
0.1	0.012028	0.009922
0.2	0.029524	0.033452
0.3	0.058813	0.074460
0.4	0.106682	0.132594
0.5	0.175843	0.214499
0.6	0.268876	0.318250
0.7	0.395747	0.448431
0.8	0.563209	0.602994
0.9	0.760523	0.783120
1.0	1.000000	1.000000

Table D-3: Normalized elevation and normalized volume values for Black Hawk Lake

	1916	1935	1973
h/H	v/V	v/V	v/V
0.0	0.011874	0.010236	0.001932
0.1	0.045293	0.050052	0.004530
0.2	0.108162	0.112118	0.010994
0.3	0.174994	0.189321	0.054338
0.4	0.263956	0.276971	0.131377
0.5	0.362612	0.373205	0.234736
0.6	0.470090	0.477696	0.358852
0.7	0.586172	0.590746	0.498247
0.8	0.711628	0.713221	0.651046
0.9	0.848274	0.847730	0.817404
1.0	1.000000	1.000000	1.000000

Table D-4: Equations used for NENV model

h/H	v^{**}_i/V	$v^{*}_{i,i-1}$	v^{**}_i
0	x_0	$\frac{\Delta H}{2} a_0 (assumed)$	$\frac{\Delta H}{2} a_0$
0.1	x_1	$\frac{\Delta H}{2}(a_1 + a_0)$	$\frac{\Delta H}{2}(a_1 + 2a_0)$
0.2	x_2	$\frac{\Delta H}{2}(a_2 + a_1)$	$\frac{\Delta H}{2}(a_2 + 2\sum_0^1 a_i)$
0.3	x_3	$\frac{\Delta H}{2}(a_3 + a_2)$	$\frac{\Delta H}{2}(a_3 - 2\sum_0^2 a_i)$
0.4	x_4	$\frac{\Delta H}{2}(a_4 + a_3)$	$\frac{\Delta H}{2}(a_4 + 2\sum_0^3 a_i)$
0.5	x_5	$\frac{\Delta H}{2}(a_5 + a_4)$	$\frac{\Delta H}{2}(a_5 - 2\sum_0^4 a_i)$
0.6	x_6	$\frac{\Delta H}{2}(a_6 + a_5)$	$\frac{\Delta H}{2}(a_6 + 2\sum_0^5 a_i)$
0.7	x_7	$\frac{\Delta H}{2}(a_7 + a_6)$	$\frac{\Delta H}{2}(a_7 + 2\sum_0^6 a_i)$
0.8	x_8	$\frac{\Delta H}{2}(a_8 + a_7)$	$\frac{\Delta H}{2}(a_8 + 2\sum_0^7 a_i)$
0.9	x_9	$\frac{\Delta H}{2}(a_9 + a_8)$	$\frac{\Delta H}{2}(a_9 + 2\sum_0^8 a_i)$
1.0	1.0	$\frac{\Delta H}{2}(A + a_9)$	$\frac{\Delta H}{2}(A + 2\sum_0^9 a_i)$

COMPUTER PROGRAM PRINTOUT

```
c
      integer i
      real x(0:20),a(0:20),aa,h,sum
      real term,sum1
      real deltah

      print*, 'input value A'
      read(*,*) aa

c      aa = 68.44

      print*,'input value of a(0)'
      read(*,*) a(0)


c      a(0) = 0.05*aa
      sum = 0.
      sum1 = 0.

      print*, 'input value H'
      read(*,*) h


c      h = 14.4

      deltah = h*0.1

      do i = 0,9
         print*, 'Input the value of x',i
         read(*,*) x(i)
      enddo

c      x(0) = 0.0041
c      x(1) = 0.0218
c      x(2) = 0.0527
c      x(3) = 0.1001
c      x(4) = 0.1705
c      x(5) = 0.2610
c      x(6) = 0.3690
c      x(7) = 0.4943
c      x(8) = 0.6397
c      x(9) = 0.8071

      sum = x(9)-x(0)
      do i = 8,1,-1
         sum = sum - ((-1)**i)*2.*x(i)
      enddo
      sum1 = sum
      sum1 = sum1 + x(9) -1

      sum = sum*aa/(1.-x(9))
      sum = (a(0)-sum)*(1.-x(9))
      a(9) = sum/sum1
```

```
do i = 1,8
   sum = 0.
   sum1 = 0.
   do j = 8,i+1,-1
      sum = sum - ((-1)**(i+j))*2.*x(j)
   enddo
   sum1 = sum
   sum = sum + ((-1)**i)*x(9) - x(i)
   sum1 = sum1 + ((-1)**i)* (2.*x(9)-1.) - x(i)
   term = (1.- x(9))


   sum = sum*aa/term
   sum1 = sum1*a(9)/term
   a(i) = sum + sum1

enddo


print*, 'Outputting values of a'
sum = 0.
do i = 0,9
print*,i,a(i)
sum = sum + a(i)
enddo


vol = deltah*(aa+2.*sum-a(0))/2.
print*, 'volume = ', vol

stop
end
```

NENV Algorithm

As mentioned earlier in Chapter 5 of the thesis to compute the volume of the lake it is necessary to solve for a_1, a_2, ..., a_9. In order to achieve this objective, it is necessary to express these variables in terms of some known values.

From Table D-4,

$$x_9 = \frac{x_9}{1} = \frac{a_9 + 2\sum_{0}^{8} a_i}{A + 2\sum_{0}^{9} a_i} \qquad (1)$$

It is possible using "dividendo", to represent the arithmetic series $a_0+a_1+\ldots+a_8$ in terms of A and a_9

According to "dividendo" if;

$$\frac{a}{b} = \frac{c}{d}$$

then,

$$\frac{a}{b-a} = \frac{c}{d-c} \qquad (A)$$

Verification of "dividendo"

Cross-multiplying the terms in (A) we get,

$$a(d-c)=c(b-a)$$

Expanding we get,

$$ad-ac=bc-ac$$

Cancelling the common term ‘ac’ we get,

$$ad=bc$$

or,

$$\frac{a}{b}=\frac{c}{d}$$

Thus by "divendo"

$$\frac{x_9}{1-x_9}=\frac{a_9+2\sum_0^8 a_i}{A+[(2\sum_0^8 a_i)+2a_9]-a_9-2\sum_0^8 a_i}$$

Thus,

$$\frac{x_9}{1-x_9}=\frac{a_9+2\sum_0^8 a_i}{A+a_9} \qquad (2)$$

Multiplying both sides of equation by (A+a_9) we get,

$$\frac{x_9}{1-x_9}(A+a_9)=a_9+2\sum_0^8 a_i$$

Combining the terms for A and a_9 we get,

$$2\sum_0^8 a_i=[(\frac{x_9}{1-x_9})A+(\frac{x_9}{1-x_9}-1)a_9]$$

Solving for the arithmetic series $a_0+a_1+\ldots+a_8$ we get,

$$\sum_0^8 a_i=0.5\times[\frac{x_9}{1-x_9}A+\frac{2x_9-1}{1-x_9}a_9] \tag{3}$$

Now from Table D-4,

$$x_8=\frac{a_8+2\sum_0^7 a_i}{A+2\sum_0^9 a_i}$$

By adding and substracting a_8 in the numerator, and expanding the denominator we get,

$$x_8=\frac{2\sum_0^7 a_i+a_8+(a_8-a_8)}{A+2\sum_0^8 a_i+2a_9}=\frac{2\sum_0^8 a_i-a_8}{A+2\sum_0^8 a_i+2a_9}$$

Thus

$$x_8 = \frac{a_8 + 2\sum_{0}^{7} a_i}{A + 2\sum_{0}^{9} a_i} = \frac{2\sum_{0}^{7} a_i + a_8 + (a_8 - a_8)}{A + 2\sum_{0}^{8} a_i + 2a_9} = \frac{2\sum_{0}^{8} a_i - a_8}{A + 2\sum_{0}^{8} a_i + 2a_9} \tag{4}$$

By substituting (3) in (4), we get

$$x_8 = \frac{[\frac{x_9}{1-x_9}A + \frac{2x_9-1}{1-x_9}a_9] - a_8}{A + [\frac{x_9}{1-x_9}A + \frac{2x_9-1}{1-x_9}a_9] + 2a_9} \tag{5}$$

Consider the denominator (D) of (5);

$$D = A + 2a_9 + (\frac{x_9}{1-x_9})A + (\frac{2x_9-1}{1-x_9})a_9$$

Combining terms for A and a_9 we get,

$$D = [1 + (\frac{x_9}{1-x_9})]A + [2 + \frac{(2x_9-1)}{1-x_9}]a_9$$

Taking least common denominator (L.C.D.) of both the terms we get,

$$D = (\frac{1}{1-x_9})A + (\frac{1}{1-x_9})a_9$$

Therefore,

$$D = \frac{1}{1-x_9}(A + a_9)$$

But,

$$D=A+2\sum_{0}^{9}a_i$$

Thus,

$$A+2\sum_{0}^{9}a_i=\frac{1}{1-x_9}(A+a_9) \tag{B}$$

Substituting (B) in (5) we get,

$$x_8=\frac{(\frac{x_9}{1-x_9})A+(\frac{2x_9-1}{1-x_9})a_9-a_8}{(\frac{1}{1-x_9})(A+a_9)}$$

Cross-multiplying we get,

$$\frac{x_8}{1-x_9}(A+a_9)=(\frac{x_9}{1-x_9})A+(\frac{2x_9-1}{1-x_9})a_9-a_8$$

Solving for a_8 and combining the terms we get,

$$a_8=[\frac{x_9-x_8}{1-x_9}A+\frac{-x_8+2x_9-1}{1-x_9}a_9]$$

Thus we have represented a_8 in terms of A and a_9. Next, we will represent a_7 in terms of A and a_9.

From Table D-4 we have,

$$x_7 = \frac{a_7 + 2\sum_{0}^{6} a_i}{A + 2\sum_{0}^{9} a_i}$$

Thus,

$$x_7 = \frac{a_7 + 2\sum_{0}^{8} a_i - 2a_8 - 2a_7}{A + 2\sum_{0}^{9} a_i} = \frac{-a_7 + 2\sum_{0}^{8} a_i - 2a_8}{A + 2\sum_{0}^{9} a_i} \qquad \text{(C)}$$

Substituting (3), (6), and (B) in (C) we get,

$$x_7 = \frac{-a_7 + [(\frac{x_9}{1-x_9})A + (\frac{2x_9-1}{1-x_9})a_9] - 2[(\frac{x_9-x_8}{1-x_9})A + (\frac{2x_9-x_8-1}{1-x_9})a_9]}{(\frac{1}{1-x_9})(A+a_9)}$$

By cross-multiplying and expanding we get,

$$\frac{x_7}{1-x_9}(A+a_9) = -a_7 + (\frac{x_9}{1-x_9})A + (\frac{2x_9-1}{1-x_9})a_9 + (\frac{-2x_9+2x_8}{1-x_9})A + (\frac{-4x_9+2x_8+2}{1-x_9})a_9$$

Combining the terms for A and a_9 and solving for a_7 we get,

$$a_7=[\frac{-x_9+2x_8-x_7}{1-x-9}A+\frac{-x_7+2x_8-2x_9+1}{1-x_9}a_9] \tag{7}$$

Thus we have represented a_7 in terms of A and a_9

By following the procedure similar to the one outlined above we can represent a_7, a_6, ..., a_0, in terms of A and a_9

For example,

$$a_6=[\frac{x_9-2x_8+2x_7-x_6}{1-x_9}A+\frac{-x_6+2x_7-2x_8+2x_9-1}{1-x_9}a_9] \tag{8}$$

and

$$a_0=[\frac{x_9-2x_8+2x_7-2x_6+2x_5-2x_4+2x_3-2x_2+2x_1-x_0}{1-x_9}A+$$

$$\frac{-x_0+2x_1-2x_2+2x_3-2x_4+2x_5-2x_6+2x_7-2x_8+2x_9-1}{1-x_9}a_9] \tag{9}$$